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BIAS EFFECTS ON MOTION STABILITY OF SUBMERSIBLE VEHICLES

by

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BIAS EFFECTS ON MOTION STABILITY OF SUBMERSIBLE VEHICLES

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This thesis analyzes the nonlinear characteristics of motion stability of a submersible vehicle in combined sway, yaw, and roll motions. Previous results, at zero pitch angles, indicate that limit cycles are generated as a result of loss of stability. In this work, these results are extended to include nonzero pitch angles. This analysis can determine how changes in vehicle parameters and loading conditions will affect its operation and performance. Stability domains are generated for a variety of vehicle and environmental parameters. A nonlinear analysis is conducted in order to assess the stability characteristics of the resulting limit cycles. The results can lead to design guidelines for improving vehicle operational envelopes.

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I. INTRODUCTION

A. DESCRIPTION OF THE PROBLEM

The study of dynamic responses in a submersible vehicle using a nonlinear analysis is important in determining operating envelopes for the vehicle. Previous studies have shown that in straight line motion, the coupling of the sway, yaw, and roll equations produce oscillating losses of stability in the system. [Ref. 5] Introducing a nonzero pitch angle to the vehicles motion will allow us to study the changes to the stability domain for a variety of environmental conditions.

In our work, the linearized equations of motion are studied using an eigenvalue analysis to determine the systems stability through a variety of operational parameters. A nonlinear analysis is then conducted to assess the stability characteristics of the resulting limit cycles and their impact on the operating characteristics of the vehicle.

B. OBJECTIVES AND OUTLINE

In this thesis we expand on previous thesis work which examined the problem of stability in straight line motions of a submersible vehicle [Ref. 13]. The primary cause for this loss of stability is the coupling between the sway/yaw/roll equations of motion for a submersible vehicle. We know that the loss of stability creates stable limit cycles in straight line motion. This work analyzes the effects of introducing a nonzero pitch angle to the generic equations of motion in order to determine their effect on the creation of limit cycles.

The model used for this work is a variant of the Swimmer Delivery Model used in [Ref. 5] for which a generic set of hydrodynamic and geometric properties are available.

II. EQUATIONS OF MOTION

A. COORDINATE SYSTEM

A moving coordinate system was used for our analysis with the origin fixed at the vehicles center of buoyancy. The *x*-axis is fixed to the longitudinal plane of symmetry for the vehicle, the *y*-axis is positive to starboard, and the *z*-axis is positive downward. All symbols used in the development of the equations of motion are summarized in Table 1.

B. GENERAL FORM OF THE EQUATIONS OF MOTION

The equations of motion for a submersible vehicle in the horizontal plane are written as follows:

<u>Sway equation:</u>

$$
\begin{aligned}
& m\left[\dot{v} + Ur - wp + x_G\left(pq + \dot{r}\right) - y_G\left(p^2 + r^2\right) + z_G\left(qr - \dot{p}\right)\right] = \\
& Y_{\dot{p}}\dot{p} + Y_{\dot{r}}\dot{r} + Y_{pq}pq + Y_{qr}qr + Y_v Uv + Y_{vw}vw + \\
& Y_{\delta_r}U^2\delta_r + Y_{\dot{v}}\dot{v} + Y_p Up + Y_r Ur + Y_{vq}vq + Y_{wp}wp + Y_{wr}wr + \\
& (W - B)\cos\theta\sin\phi - \\
& \int_{x_{tail}}^{x_{nose}} \left[C_{D_y} h(x)(v + xr)^2 + C_{D_z} b(x)(w - xq)^2\right]\frac{(v + xr)}{U_{cf}(x)}\,dx
\end{aligned}
\qquad (1)
$$

Yaw equation:

$$I_{zz}\dot{r}+\left(I_{zz}-I_{yy}\right)pq-I_{xy}\left(p^2-q^2\right)-I_{yz}\left(pr+\dot{q}\right)+I_{xz}\left(qr-\dot{p}\right)+$$
$$m\left[x_G\left(\dot{v}+Ur-wp\right)-y_G\left(\dot{U}-vr+wq\right)\right]=$$
$$N_{\dot{p}}\dot{p}+N_{\dot{r}}\dot{r}+N_{pq}pq+N_{qr}qr+N_vUv+N_{vw}vw+$$
$$N_{\delta_r}U^2\delta_r+N_{\dot{v}}\dot{v}+N_pUp+N_rUr+N_{vq}vq+N_{wp}wp+N_{wr}wr+$$
$$\left(x_GW-x_BB\right)\cos\theta\sin\phi+\left(y_GW-y_BB\right)\sin\theta+U^2N_{prop}-$$
$$\int_{x_{tail}}^{x_{nose}}\left[C_{D_y}h(x)(v+xr)^2+C_{D_z}b(x)(w-xq)^2\right]\frac{(v+xr)}{U_{cf}(x)}xdx \tag{2}$$

Roll equation:

$$I_{xx}\dot{p}+\left(I_{zz}-I_{yy}\right)qr+I_{xy}\left(pr-\dot{q}\right)-I_{yz}\left(q^2-r^2\right)-I_{xz}\left(pq+\dot{r}\right)+$$
$$m\left[y_G\left(\dot{w}-Uq+vp\right)-z_G\left(\dot{v}+Ur-wp\right)\right]=$$
$$K_{\dot{p}}\dot{p}+K_{\dot{r}}\dot{r}+K_{pq}pq+K_{qr}qr+K_vUv+K_{vw}vw+$$
$$K_{prop}U^2+K_{\dot{v}}\dot{v}+K_pUp+K_rUr+K_{vq}vq+K_{wp}wp+K_{wr}wr+$$
$$\left(y_GW-y_BB\right)\cos\theta\cos\phi-\left(z_GW-z_BB\right)\cos\theta\sin\phi \tag{3}$$

The rotational velocity equation around the x-axis:

$$\dot{\phi}=p \tag{4}$$

U_{cf} denotes the cross-flow velocity:

$$U_{cf}(x)=\sqrt{(v+xr)^2+(w-xq)^2} \tag{5}$$

C_D	quadratic drag coefficient
d_r	rudder deflection
$h(x)$	local height of hull
(I_{xx}, I_{yy}, I_{zz})	vehicle mass moments of inertia about body axes
(I_{xy}, I_{yz}, I_{zx})	cross products of inertia
(K,M,N)	moment components along the three axes
m	vehicle mass
(p,q,r)	rotational velocity components along the body axes
(f,q,y)	Euler angles
U	constant vehicle speed along the x-axis
(u,v,w)	translational velocities about (x,y,z) axes
(x,y,z)	distances along the three body axes
(X,Y,Z)	force components along the body axes
(x_G, y_G, z_G)	coordinates of the center of gravity
(x_B, y_B, z_B)	coordinates of the center of buoyancy
x_{nose}	fore coordinate of vehicle body
x_{tail}	aft coordinate of vehicle body

Table 1: Nomenclature

C. SIMPLIFICATIONS

We must simplify the equations of motion in order to reflect the fact that we are analyzing motion about the *y*-axis. The simplifications that we employ are:

- Acceleration, $\dot{w}$, in the *z*-direction is zero.
- Acceleration in the longitudinal direction, $\dot{u}$, is zero.
- Rotational velocity, q, and acceleration, $\dot{q}$, in the *y*- direction are zero

D. SIMPLIFIED EQUATIONS OF MOTION

After applying the above simplifications, the equations of motion become:

Sway equation:

$$m\left[\dot{v}+Ur-wp+x_G\dot{r}-y_G\left(p^2+r^2\right)+z_G\dot{p}\right]=$$

$$Y_{\dot{p}}\dot{p}+Y_{\dot{r}}\dot{r}+Y_vUv+Y_{vw}vw+Y_{\delta_r}U^2\delta_r+Y_{\dot{v}}\dot{v}+Y_pUp+Y_rUr+Y_{wp}wp+Y_{wr}wr+$$

$$(W-B)\cos\theta\sin\phi-$$

$$\int_{x_{tail}}^{x_{nose}}\left[C_{D_y}h(x)(v+xr)^2+C_{D_z}b(x)w^2\right]\frac{(v+xr)}{U_{cf}(x)}dx \tag{6}$$

Yaw equation:

$$I_{zz}\dot{r}-I_{xy}p^2-I_{yz}pr+I_{xz}\dot{p}+$$

$$m\left[x_G\left(\dot{v}+Ur-wp\right)-y_G\left(\dot{U}-vr\right)\right]=$$

$$N_{\dot{p}}\dot{p}+N_{\dot{r}}\dot{r}+N_vUv+N_{vw}vw+N_{\delta_r}U^2\delta_r+N_{\dot{v}}\dot{v}+N_pUp+N_rUr+N_{wp}wp+$$

$$N_{wr}wr+\left(x_GW-x_BB\right)\cos\theta\sin\phi+\left(y_GW-y_BB\right)\sin\theta+U^2N_{prop}-$$

$$\int_{x_{tail}}^{x_{nose}}\left[C_{D_y}h(x)(v+xr)^2+C_{D_z}b(x)w^2\right]\frac{(v+xr)}{U_{cf}(x)}xdx \tag{7}$$

Roll equation:

$$I_{xx}\dot{p}+I_{xy}pr-I_{yz}r^2-I_{xz}\dot{r}+m\left[y_G\left(vp\right)-z_G\left(\dot{v}+Ur-wp\right)\right]=$$

$$K_{\dot{p}}\dot{p}+K_{\dot{r}}\dot{r}+K_vUv+K_{vw}vw+K_{prop}U^2+K_{\dot{v}}\dot{v}+K_pUp+K_rUr+$$

$$K_{wp}wp+K_{wr}wr+\left(y_GW-y_BB\right)\cos\theta\cos\phi-\left(z_GW-z_BB\right)\cos\theta\sin\phi \tag{8}$$

Roll rate:

$$\dot{\phi} = p \tag{9}$$

III. LINEAR ANALYSIS

A. LINEARIZATION

The simplified equations of motion can be written in the matrix form:

$$\mathbf{A}\dot{\mathbf{x}} = \mathbf{B}\mathbf{x} + \mathbf{g}(\mathbf{x}) \qquad (10)$$

where the state vector, $\mathbf{x}$, is defined as,

$$\mathbf{x} = \begin{bmatrix} v \\ r \\ p \\ \phi \end{bmatrix}$$

and the state matrices are,

$$\mathbf{A} = \begin{bmatrix} m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} & -mz_G - Y_{\dot{p}} & 0 \\ mx_G - N_{\dot{v}} & I_{zz} - N_{\dot{r}} & -I_{xz} - N_{\dot{p}} & 0 \\ -mz_G - K_{\dot{v}} & I_{xz} - K_{\dot{r}} & I_{xx} - K_{\dot{p}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\mathbf{B} = \begin{bmatrix} Y_v + Y_{vw}w & Y_r U + Y_{wr} - mU & Y_p U + Y_{wp} w + mw & 0 \\ N_v + N_{vw}w & -mx_G U + N_r U + N_{wr} w & N_p U + N_{wp} w & 0 \\ K_v U + K_{vw} w & mz_G U + K_r U + K_{wr} w & -mz_G w + K_p U + K_{wp} w & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The **g(x)** term remains as given [Ref. 13].

The nonlinear terms are linearized around a nominal point:

$$\mathbf{x}_0 = [v_0, r_0, p_0, \phi_0]^T = 0$$

A Taylor series expansion is applied to the nonlinear terms about the nominal point, $\mathbf{x}_0$, and keeping only the linear components, the equations of motion, written in matrix form, become:

$$\mathbf{A}'\dot{\mathbf{x}} = \mathbf{B}'\mathbf{x} \tag{11}$$

where,

$$\mathbf{A}' = \mathbf{A}$$

and

$$\mathbf{B}' = \begin{bmatrix} Y_v U & Y_r U - mU & Y_p U & (W - B)\cos\theta \\ N_v U & -mx_G U + N_r U & N_p U & (x_G W - x_B B)\cos\theta \\ K_v U & mz_G U + K_r U & K_p U & (-z_G W + z_B B)\cos\theta \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

To assess the dynamic stability of the vehicle, an Eigenvalue analysis is performed in the next section.

B. LOSS OF STABILITY

An Eigenvalue analysis is used to determine the stability of the linearized system. Stability is dependent on the location of the Eigenvalues or the roots of the characteristic equation:

$$\det(B' - \lambda A' = 0) \tag{12}$$

in the polynomial form:

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \tag{13}$$

The coefficients equation (13) are given in [Ref. 13, pg. 11]. Using Routh's criterion we can examine the stability of the system. The following conditions must be applied to the characteristic equation (13) to ensure all roots have negative real parts:

$$BCD - AD^2 - EB^2 > 0 \tag{14}$$

$$E > 0 \tag{15}$$

If E is less than zero, one real root of (13) becomes positive and the system will become unstable in a divergent manner [Ref. 9]. This is the case of a directionally unstable ship which is well known in the literature [Ref. 3]. If the condition in (14) is not met, it means that there is at least one complex conjugate root with real parts and will result in an oscillatory response, indicating loss of stability.

To analyze the limiting case of loss of stability equation (14) must be solved in the following form:

$$BCD - AD^2 - EB^2 = 0 \tag{16}$$

The result of this equation will be a curve of z_G as a function of x_G and will be our locus for loss of stability. We can express the coefficients of equation (16) in the form:

$$A = A_1 z_G^2 + A_2 z_G + A_3 \tag{17}$$

$$B = B_1 z_G^2 + B_2 z_G + B_3 \tag{18}$$

A_1, A_2, and A_3 are of the form given in [Ref. 13, pg. 14].

B_1 and B_3 are of the form given in [Ref. 13, pg. 14]. With the addition of a pitch angle, w,

B_2 takes the form:

$$\begin{aligned}
B_2 = &-m(K_v U)(I_{zz} - N_{\dot r}) - m(I_{zz} - N_{\dot r})(Y_p U) \\
&+ mY_{\dot p}(UN_r - Umx_G) + mK_{\dot v}(UN_r - Umx_G) \\
&- m(N_{\dot p} + I_{xz})(UY_r - Um) + m(N_p U)(mx_G - Y_{\dot r}) \\
&- m(K_{\dot r} - I_{xz})(N_v U) + mUY_{\dot p}(mx_G - N_{\dot v}) \\
&- mU(N_{\dot p} + I_{xz})(m - Y_{\dot v}) + mUK_r(mx_G - N_{\dot v}) \\
&+ mz_G w(m - Y_{\dot v})(I_{zz} - N_{\dot r}) - mz_G w(mx_G - Y_{\dot r})(mx_G - N_{\dot v})
\end{aligned}$$

$$C = C_1 z_G^2 + C_2 z_G + C_3 \tag{19}$$

C_1 and C_3 are of the form given in [Ref. 13, pg. 15]. With the addition of a pitch angle, w, C_2 takes the form:

$$\begin{aligned}
C_2 = & \, mU(mx_G - N_{\dot{v}})(Y_pU) - mUY_{\dot{p}}(N_vU) - mUK_r(N_vU) \\
& + mU(N_{\dot{p}} + I_{xz})(Y_vU) - mU(N_pU)(m - Y_{\dot{v}}) \\
& + W(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - W(mx_G - Y_{\dot{r}})(mx_G - N_{\dot{v}}) \\
& - m(X_BB - x_GW)(mx_G - Y_{\dot{r}}) + m(UN_r - Umx_G)(Y_pU) \\
& - m(UY_r - Um)(N_pU) + m(K_vU)(UN_r - Umx_G) \\
& - mz_Gw(m - Y_{\dot{v}})(UN_r - Umx_G) - mz_Gw(Y_vU)(I_{zz} - N_{\dot{r}}) \\
& - mz_Gw(mx_G - Y_{\dot{r}})(N_vU) + mz_Gw(mx_G - N_{\dot{v}})(UY_r - Um)
\end{aligned}$$

$$D = D_1 z_G + D_2 \tag{20}$$

D_1 is of the form given in [Ref. 13, pg. 16]. With the addition of a pitch angle, w, D_2 takes the form:

$$\begin{aligned}
D_2 = & \, UK_r(x_BB - x_GW)(m - Y_{\dot{v}}) - UK_r(N_vU)(Y_pU) \\
& + UK_r(N_pU)(Y_vU) - (K_{\dot{r}} - I_{xz})(x_BB - x_GW)(Y_pU) \\
& + K_{\dot{v}}(x_BB - x_GW)(UY_r - Um) - (K_vU)(x_BB - x_GW)(mx_G - Y_{\dot{r}}) \\
& + (K_vU)(UN_r - Umx_G)(Y_pU) - (K_vU)(UY_r - Um)(N_pU) \\
& - (K_pU)(Y_vU)(UN_r - Umx_G) + (K_pU)(UK_r - Um)(N_vU) \\
& + mz_Gw(Y_vU)(UN_r - Umx_G) - mz_gw(UK_r - Um)(N_vU)
\end{aligned}$$

The equation for the coefficient, E, remains unchanged [Ref. 13, pg. 16]. Applying the stability criterion, equation (16), and utilizing them in the resulting fifth order polynomial, F, [Ref. 13, pg. 18] we are able to solve F using the MATLAB program in Appendix A. The curves show z_G as a function of x_G and we show results for varying

pitch angles, w, and varying forward velocities, U. On all of the following graphs the pitch angle is varied from 10 degrees to -10 degrees in increments of five degrees. The top curve is the 10 degree curve and the bottom curve is the -10 degree curve.

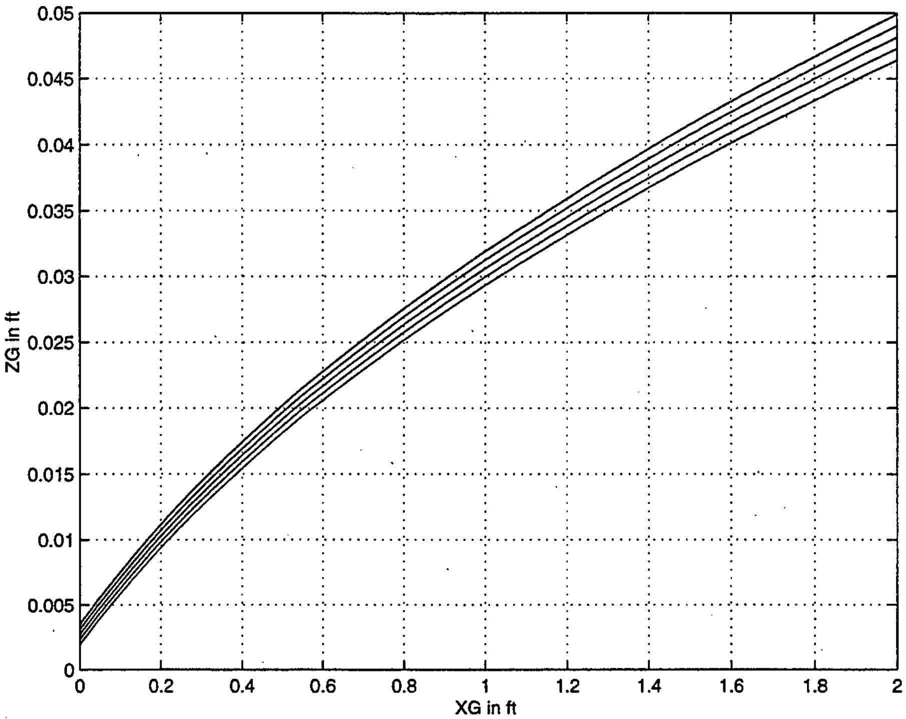

Figure 1: ZG vs. XG for U = 2 ft/sec

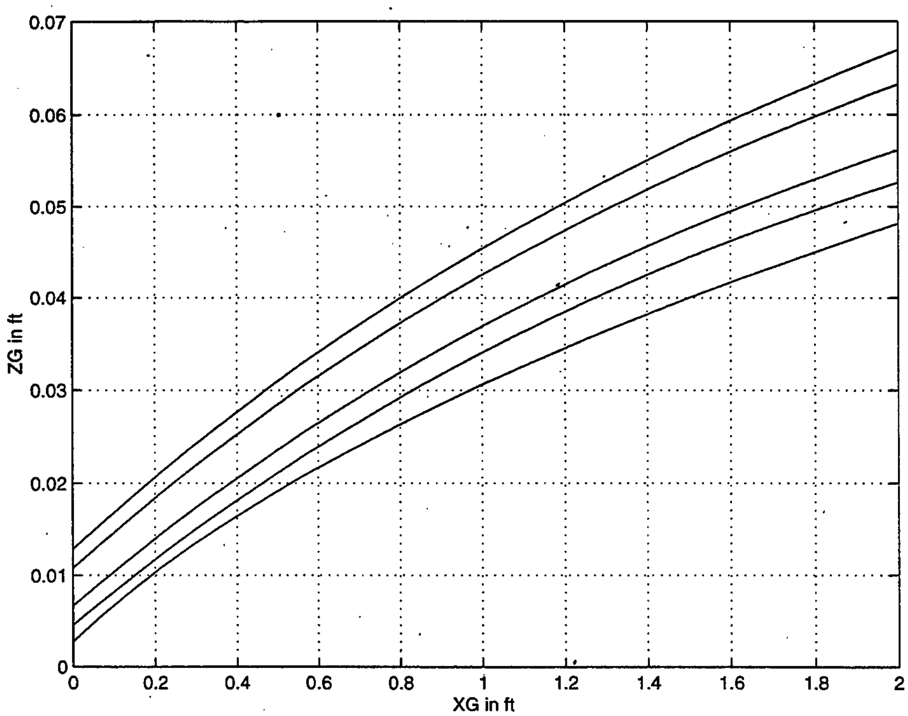

Figure 2: ZG vs. XG for U = 3.5 ft/sec

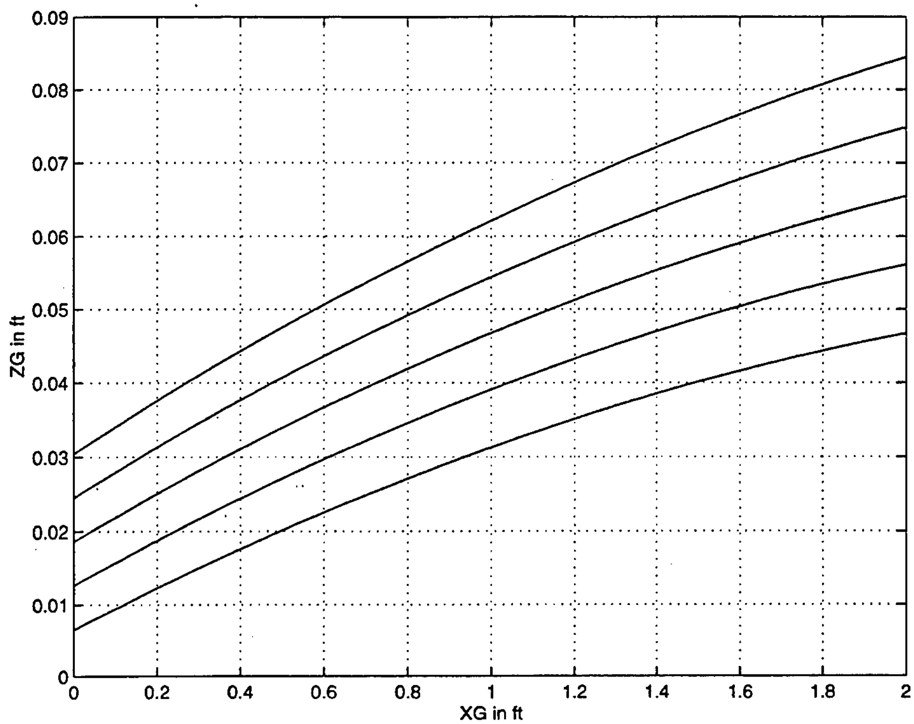

Figure 3: ZG vs. XG for U = 5 ft/sec

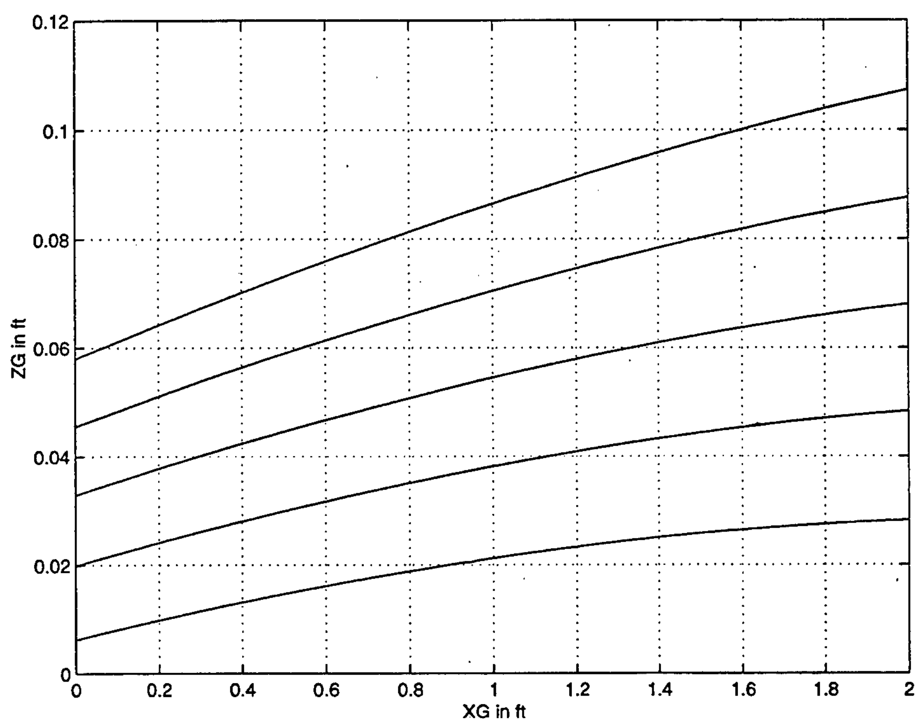

Figure 4: ZG vs. XG for U = 6.5 ft/sec

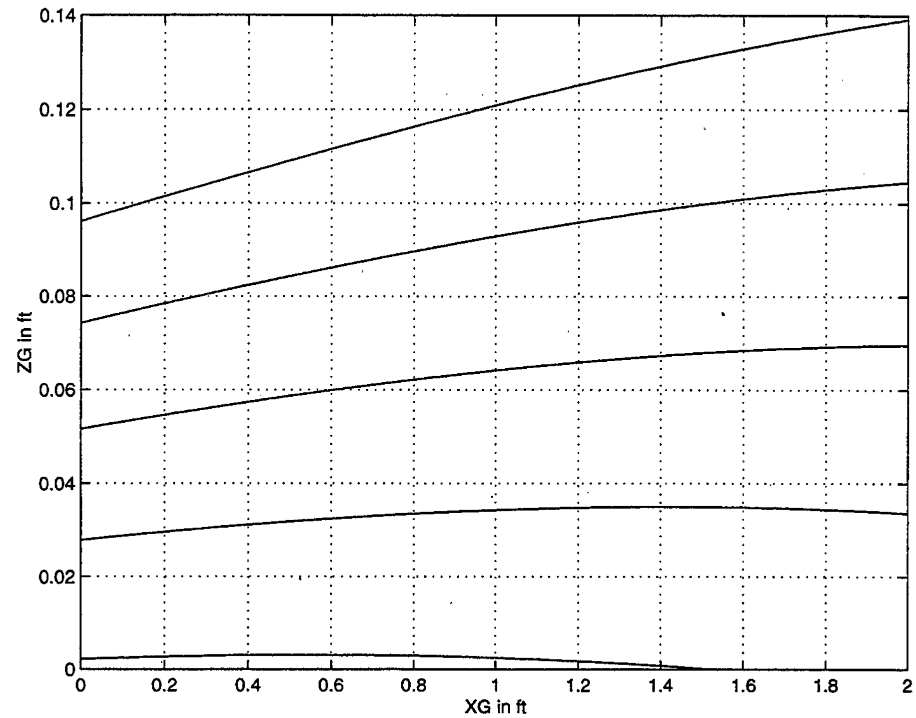

Figure 5: ZG vs. XG for U = 8 ft/sec

From the results of Figures 1 through 5, the following conclusions can be drawn:

1. In all cases, a sufficiently high metacentric height is required in order to ensure vehicle stability. Regions of parameters that fall below the critical curves correspond to dynamic instability.
2. The critical metacentric height that is required for dynamic stability is an increasing function of both vehicle speed and trim angle.
3. Static stability alone does not necessarily ensure dynamic stability of motion during the turn.
4. The loss of stability experienced here is a dynamic loss of stability. At the critical metacentric height, one pair of complex conjugate eigenvalues possesses a zero real part. This is an oscillatory loss of stability, which can not be predicted by considering the uncoupled sway/yaw equations of motion alone.

IV. NONLINEAR ANALYSIS

A. INTRODUCTION

In the previous chapter we performed a linear analysis of the equations of motion, observing that with changes to specific parameters (x_G, z_G, w) it is possible to pass through a stable region to a region of loss of stability.

In previous work [Ref 13] it was shown that these bifurcations to periodic solutions were all supercritical. This means that limit cycles were produced after the loss of stability. By introducing a pitch angle, w, in the nonlinear analysis we will analyze whether the bifurcations to periodic solutions will remain supercritical and what changes occur to the limit cycles themselves.

B. THIRD ORDER EXPANSIONS

Our linearized system was written in the form of equation (11), ignoring the nonlinear terms. Including the nonlinear terms changes the form of equation (11) to:

$$\mathbf{A}'\dot{\mathbf{x}} = \mathbf{B}'\mathbf{x} + \mathbf{g}(\mathbf{x}) \tag{21}$$

where:

$$\mathbf{g}(\mathbf{x})=\begin{bmatrix} g_1(x) \\ g_2(x) \\ g_3(x) \\ g_4(x) \end{bmatrix}$$

Keeping terms up to third order, the matrix **g(x)** can be written in the vector form:

$$g(x)=g^{(2)}(x)+g^{(3)}(x)+c(x) \tag{22}$$

where $g^{(2)}(x)$ contains the second order nonlinear terms and $g^{(3)}(x)$ contains the third order nonlinear terms. The cross flow integrals and the second order nonlinear terms remain unchanged with the addition of a pitch angle, w. [Ref. 13, pg.22, 23] However the third order nonlinear terms take the form:

$$\mathbf{g}(\mathbf{x})=\begin{bmatrix} g_1^{(3)}(x) \\ g_2^{(3)}(x) \\ g_3^{(3)}(x) \\ g_4^{(3)}(x) \end{bmatrix}$$

where:

$$g_1^{(3)}=-I_v^{(3)}-\frac{1}{6}(W-B)\phi^3\cos\theta$$

$$g_2^{(3)}=-I_r^{(3)}-\frac{1}{6}(x_g W-x_B B)\phi^3\cos\theta$$

$$g_3^{(3)}=\frac{1}{6}(z_g W-z_B B)\phi^3\cos\theta$$

$$g_4^{(3)}=0$$

The Taylor series expansion yielding the second and third order linear terms of the cross flow integrals, and the inverse of the system matrix, $(A')^{-1}$, remain unchanged. [Ref. 13,

pg. 24, 25]. However the *B'* matrix has changed as shown in the Linear Analysis section of this work. These changes to the 4 x 4 *F* matrix are:

$$\mathbf{F} = \begin{bmatrix} \frac{F_{11}}{D} & \frac{F_{12}}{D} & \frac{F_{13}}{D} & \frac{F_{14}}{D} \\ \frac{F_{21}}{D} & \frac{F_{22}}{D} & \frac{F_{23}}{D} & \frac{F_{24}}{D} \\ \frac{F_{31}}{D} & \frac{F_{32}}{D} & \frac{F_{33}}{D} & \frac{F_{34}}{D} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

where:

$$F_{11} = a_{11}(Y_v U) + a_{12}(N_v U) + a_{13}(K_v U)$$
$$F_{12} = a_{11}(Y_r - m)U + a_{12}(N_r - mx_G)U + a_{13}(K_r + mz_G)U$$
$$F_{13} = a_{11}(Y_p U) + a_{12}(N_{pv} U) + a_{13}(K_{pv} U)$$
$$F_{14} = a_{11}(W - B)\cos\theta + a_{12}(x_G W - x_B B)\cos\theta + a_{13}(-z_G W + z_B B)\cos\theta$$
$$F_{21} = a_{21}(Y_v U) + a_{22}(N_v U) + a_{23}(K_v U)$$
$$F_{22} = a_{21}(Y_v - m)U + a_{22}(N_v - mx_G)U + a_{23}(K_v + mz_G)U$$
$$F_{23} = a_{21}(Y_p U) + a_{22}(N_{pv} U) + a_{23}(K_{pv} U)$$
$$F_{24} = a_{21}(W - B)\cos\theta + a_{22}(x_G W - x_B B)\cos\theta + a_{23}(-z_G W + z_B B)\cos\theta$$
$$F_{31} = a_{31}(Y_v U) + a_{32}(N_v U) + a_{33}(K_v U)$$
$$F_{32} = a_{31}(Y_r - m)U + a_{32}(N_r - mx_G)U + a_{33}(K_r + mz_G)U$$
$$F_{33} = a_{31}(Y_p U) + a_{32}(N_p U) + a_{33}(K_p U)$$
$$F_{34} = a_{31}(W - B)\cos\theta + a_{32}(x_G W - x_B B)\cos\theta + a_{33}(-z_G W + z_B B)\cos\theta$$

The remaining elements of the nonlinear and constant terms remain unchanged [Ref . 13, pg. 28, 29].

C. COORDINATE TRANSFORMATIONS

To continue our analysis it is necessary to bring our transform our coordinate system from state space to a normal coordinate system. This transformation is performed in the manner given [Ref. 13, pg. 29, 30].

D. CENTER MANIFOLD EXPANSIONS

The center manifold expressions are of the form given [Ref. 13, pg. 30-33]. However, the coefficients l_{ij}, i=1, 2, 3; j=5, 6, 7 are:

$$\begin{aligned} l_{1,5} &= \frac{a_{11}}{D} y_G\left(m_{31}^2 + m_{21}^2\right) \\ &+ \frac{a_{12}}{D}\left(I_{xy} m_{31}^2 + I_{yz} m_{31} m_{21} + y_G m_{21} m_{11}\right) \\ &- \frac{a_{13}}{D}\begin{pmatrix} I_{xy} m_{31} m_{21} + I_{yz} m_{21}^2 + m y_G m_{11} m_{31} \\ + y_g W m_{41}^2 - y_B B m_{41}^2 \end{pmatrix} \end{aligned} \tag{23}$$

$$\begin{aligned} l_{1,6} &= \frac{a_{11}}{D} y_G\left(2 m_{31} m_{32} + 2 m_{21} + m_{22}\right) \\ &+ \frac{a_{12}}{D}\begin{bmatrix} 2 I_{xy} m_{31} m_{32} + I_{yz}\left(m_{31} m_{22} + m_{32} m_{21}\right) \\ + y_G\left(m_{21} m_{12} + m_{22} m_{11}\right) \end{bmatrix} \\ &- \frac{a_{13}}{D}\begin{bmatrix} I_{xy}\left(m_{31} m_{22} + m_{32} m_{21}\right) + 2 I_{yz} m_{21} m_{22} \\ + m y_G\left(m_{11} m_{32} + m_{12} m_{31}\right) + 2\left(y_G W - y_B B\right) m_{41} m_{42} \end{bmatrix} \end{aligned} \tag{24}$$

$$l_{1,7} = \frac{a_{11}}{D} y_G \left(m_{32}^2 + m_{22}^2\right)$$

$$+ \frac{a_{12}}{D}\left(I_{xy} m_{32}^2 + I_{yz} m_{32} m_{22} + y_G m_{22} m_{12}\right)$$

$$- \frac{a_{13}}{D}\left[I_{xy} m_{32} m_{22} + I_{yz} m_{22}^2 + m y_G m_{12} m_{32} + \left(y_G W - y_b B\right) m_{42}^2\right] \tag{25}$$

$$l_{2,5} = \frac{a_{21}}{D} y_G \left(m_{31}^2 + m_{21}^2\right)$$

$$+ \frac{a_{22}}{D}\left(I_{xy} m_{31}^2 + I_{yz} m_{31} m_{21} + y_G m_{21} m_{11}\right)$$

$$- \frac{a_{23}}{D}\begin{pmatrix} I_{xy} m_{31} m_{21} + I_{yz} m_{21}^2 + m y_G m_{11} m_{31} \\ + y_G W m_{41}^2 - y_B B m_{41}^2 \end{pmatrix} \tag{26}$$

$$l_{2,6} = \frac{a_{21}}{D} y_G \left(2 m_{31} m_{32} + 2 m_{21} + m_{22}\right)$$

$$+ \frac{a_{22}}{D}\begin{bmatrix} 2 I_{xy} m_{31} m_{32} + I_{yz}\left(m_{31} m_{22} + m_{32} m_{21}\right) \\ + y_G \left(m_{21} m_{12} + m_{22} m_{11}\right) \end{bmatrix}$$

$$- \frac{a_{23}}{D}\begin{bmatrix} I_{xy}\left(m_{31} m_{22} + m_{32} m_{21}\right) + 2 I_{yz} m_{21} m_{22} \\ + m y_G \left(m_{11} m_{32} + m_{12} m_{21}\right) + 2\left(y_G W - y_B B\right) m_{41} m_{42} \end{bmatrix} \tag{27}$$

$$l_{2,7} = \frac{a_{21}}{D} y_G \left(m_{32}^2 + m_{22}^2\right)$$

$$+ \frac{a_{22}}{D}\left(I_{xy} m_{32}^2 + I_{yz} m_{32} m_{22} + y_G m_{22} m_{12}\right)$$

$$- \frac{a_{23}}{D}\left[I_{xy} m_{32} m_{22} + I_{yz} m_{22}^2 + m y_G m_{12} m_{32} + \left(y_G W - y_B B\right) m_{42}^2\right] \tag{28}$$

$$l_{3,5} = \frac{a_{31}}{D} y_G \left(m_{31}^2 + m_{21}^2\right)$$

$$+ \frac{a_{32}}{D}\left(I_{xy} m_{31}^2 + I_{yz} m_{31} m_{21} + y_G m_{21} m_{11}\right)$$

$$-\frac{a_{33}}{D}\left(I_{xy}m_{31}m_{21}+I_{yz}m_{21}^{2}+my_{G}m_{11}m_{31}+y_{G}Wm_{41}^{2}-y_{B}Bm_{41}^{2}\right) \tag{29}$$

$$l_{3,6}=\frac{a_{31}}{D}y_{G}\left(2m_{31}m_{32}+2m_{21}+m_{22}\right)$$

$$+\frac{a_{32}}{D}\begin{bmatrix}2I_{xy}m_{31}m_{32}+I_{yz}\left(m_{31}m_{22}+m_{32}m_{21}\right)\\+y_{G}\left(m_{21}m_{12}+m_{22}m_{11}\right)\end{bmatrix}$$

$$-\frac{a_{33}}{D}\begin{bmatrix}I_{xy}\left(m_{31}m_{22}+m_{32}m_{21}\right)+2I_{yz}m_{21}m_{22}\\+my_{G}\left(m_{11}m_{32}+m_{12}m_{31}\right)+2\left(y_{G}W-y_{B}B\right)m_{41}m_{42}\end{bmatrix} \tag{30}$$

$$l_{3,7}=\frac{a_{31}}{D}y_{G}\left(m_{32}^{2}+m_{22}^{2}\right)$$

$$+\frac{a_{32}}{D}\left(I_{xy}m_{32}^{2}+I_{yz}m_{32}m_{22}+y_{G}m_{22}m_{12}\right)$$

$$-\frac{a_{33}}{D}\left[I_{xy}m_{32}m_{22}+I_{yz}m_{22}^{2}+my_{G}m_{12}m_{32}+\left(y_{G}W-y_{B}B\right)m_{42}^{2}\right] \tag{31}$$

E. AVERAGING

The procedure for averaging the equations up to the third order is the same one given in [Ref. 13, pg. 37-40]. The addition of a nonzero pitch angle yields new coefficients l_{ij}, I=1, 2, 3; j=1, 2, 3, 4. They are:

$$l_{11} = -\frac{a_{11}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(E_0 m_{11}^3 + 3E_1 m_{11}^2 m_{21} + 3E_2 m_{11} m_{21}^2 + E_3 m_{21}^3\right) \\ +\frac{1}{6}(W - B)\cos\theta m_{41}^3\end{array}\right]$$

$$-\frac{a_{12}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(E_1 m_{11}^3 + 3E_2 m_{11}^2 m_{21} + 3E_3 m_{11} m_{21}^2 + E_4 m_{21}^3\right) \\ +\frac{1}{6}(x_G W - x_B B)\cos\theta m_{41}^3\end{array}\right]$$

$$+\frac{a_{13}}{6D}(z_G W - z_B B)\cos\theta m_{41}^3 \tag{32}$$

$$l_{12} = -\frac{a_{11}}{D}\left[\frac{C_{D_y}}{6\gamma}\begin{pmatrix}3E_0 m_{11}^2 m_{12} + 3E_1\left(m_{11}^2 m_{22} + 2m_{11}m_{12}m_{21}\right) \\ +3E_2\left(m_{12}m_{21}^2 + 2m_{21}m_{22}m_{11}\right) + 3E_3 m_{21}^2 m_{22}\end{pmatrix} + \frac{1}{6}(W - B)\cos\theta 3m_{41}^2 m_{42}\right]$$

$$-\frac{a_{12}}{D}\left[\begin{array}{l}\frac{C_{D_Y}}{6\gamma}\begin{pmatrix}3E_1 m_{11}^2 m_{12} + 3E_2\left(m_{11}^2 m_{22} + 2m_{11}m_{12}m_{21}\right) + 3E_{\#}\left(m_{12}m_{21}^2 + 2m_{21}m_{22}m_{11}\right) \\ +3E_4 m_{21}^2 m_{22}\end{pmatrix} \\ +\frac{1}{6}(x_G W - x_B B)\cos\theta 3m_{41}^2 m_{42}\end{array}\right]$$

$$+\frac{a_{13}}{6D}(z_G W - z_b B)\cos\theta 3m_{41}^2 m_{42} \tag{33}$$

$$l_{13} = -\frac{a_{11}}{D}\left[\begin{array}{l}\frac{C_{D_Y}}{6\gamma}\begin{pmatrix}3E_0 m_{11} m_{12}^2 + 3E_1\left(m_{12}^2 m_{21} + 2m_{11}m_{12}m_{22}\right) + 3E_2\left(m_{22}^2 m_{11} + 2m_{21}m_{22}m_{12}\right) \\ +3E_3 m_{21} m_{22}^2\end{pmatrix} \\ +\frac{1}{6}(W - B)\cos\theta 3m_{41} m_{42}^2\end{array}\right]$$

$$-\frac{a_{12}}{D}\left[\frac{C_{D_y}}{6\gamma}\left(E_1 m_{12}^3+3E_2 m_{11}^2 m_{21}+3E_3 m_{12} m_{22}^2+E_4 m_{22}^2\right)+\frac{1}{6}\left(x_G W-x_B B\right)\cos\theta m_{42}^3\right]$$

$$+\frac{a_{13}}{6D}\left(z_G W-z_B B\right)\cos\theta m_{42}^3 \tag{34}$$

$$l_{21}=-\frac{a_{21}}{D}\left[\frac{C_{D_y}}{6\gamma}\left(E_0 m_{11}^3+3E_1 m_{11}^2 m_{21}+3E_2 m_{11} m_{21}^2+E_3 m_{21}^2\right)+\frac{1}{6}\left(W-B\right)\cos\theta m_{41}^3\right]$$

$$-\frac{a_{22}}{D}\left[\frac{C_{D_y}}{6\gamma}\left(E_1 m_{11}^3+3E_2 m_{11}^2 m_{21}+3E_3 m_{11} m_{21}^2+E_4 m_{21}^3\right)+\frac{1}{6}\left(x_G W-x_B B\right)\cos\theta m_{41}^3\right] \tag{35}$$

$$l_{22}=-\frac{a_{12}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(\begin{array}{l}3E_0 m_{11}^2 m_{12}+3E_1\left(m_{11}^2 m_{22}+2m_{11}m_{12}m_{21}\right)+3E_2\left(m_{12}m_{21}^2+2m_{21}m_{22}m_{11}\right)\\+3E_3 m_{21}^2 m_{22}\end{array}\right)\\+\frac{1}{6}\left(W-B\right)\cos\theta 3m_{41}^2 m_{42}\end{array}\right]$$

$$-\frac{a_{22}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(\begin{array}{l}3E_1 m_{11}^2 m_{12}+3E_2\left(m_{11}^2 m_{22}+2m_{11}m_{12}m_{21}\right)+3E_3\left(m_{12}m_{21}^2+2m_{21}m_{22}m_{11}\right)\\+3E_4 m_{21}^2 m_{22}\end{array}\right)\\+\frac{1}{6}\left(x_G W-x_B B\right)\cos\theta 3m_{41}^2 m_{42}\end{array}\right]$$

$$+\frac{a_{23}}{6D}\left(z_g W-z_B B\right)\cos\theta 3m_{41}^2 m_{42} \tag{36}$$

$$l_{23}=-\frac{a_{21}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(\begin{array}{l}3E_0 m_{11} m_{12}+3E_1\left(m_{12}^2 m_{21}+2m_{11}m_{12}m_{22}\right)+3E_2\left(m_{22}^2 m_{11}+2m_{21}m_{22}m_{12}\right)\\+3E_3 m_{21} m_{22}^2\end{array}\right)\\+\frac{1}{6}\left(W-B\right)\cos\theta 3m_{41} m_{42}^2\end{array}\right]$$

$$-\frac{a_{22}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(\begin{array}{l}3E_1 m_{11} m_{12}^2+3E_2\left(m_{12}^2 m_{21}+2m_{11}m_{12}m_{22}\right)+3E_3\left(m_{22}^2 m_{11}+2m_{21}m_{22}m_{12}\right)\\+3E_4 m_{21} m_{22}^2\end{array}\right)\\+\frac{1}{6}\left(x_G W-x_B B\right)\cos\theta 3m_{41} m_{42}^2\end{array}\right]$$

$$+\frac{a_{23}}{D}\left(z_G W - z_B B\right)\cos\theta 3 m_{41} m_{42}^2 \tag{37}$$

$$l_{24} = -\frac{a_{21}}{D}\left[\frac{C_{D_y}}{6\gamma}\left(E_0 m_{12}^3 + 3E_1 m_{11}^2 m_{21} + 3E_2 m_{12} m_{22}^2 + E_3 m_{22}^3\right) + \frac{1}{6}\left(W - B\right)\cos\theta m_{42}^3\right]$$

$$-\frac{a_{22}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(E_1 m_{12}^3 + 3E_2 m_{11}^2 m_{21} + 3E_3 m_{12} m_{22}^2 + E_4 m_{22}^3\right) \\ +\frac{1}{6}\left(x_G W - x_B B\right)\cos\theta m_{42}^3\end{array}\right]$$

$$+\frac{a_{23}}{6D}\left(z_G W - z_B B\right)\cos\theta m_{42}^3 \tag{38}$$

$$l_{31} = -\frac{a_{31}}{D}\left[\frac{C_{D_y}}{6\gamma}\left(E_0 m_{11}^3 + 3E_1 m_{11}^2 m_{21} + 3E_2 m_{11} m_{21}^2 + E_3 m_{21}^3\right) + \frac{1}{6}\left(W - B\right)\cos\theta m_{41}^3\right]$$

$$-\frac{a_{32}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(E_1 m_{11}^3 + 3E_2 m_{11}^2 m_{21} + 3E_3 m_{11} m_{21}^2 + E_4 m_{21}^3\right) \\ +\frac{1}{6}\left(x_G W - x_B B\right)\cos\theta m_{41}^3\end{array}\right]$$

$$+\frac{a_{33}}{6D}\left(z_G W - z_B B\right)\cos\theta m_{41}^3 \tag{39}$$

$$l_{32} = -\frac{a_{31}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(\begin{array}{l}3E_0 m_{11}^2 m_{12} + 3E_1\left(m_{11}^2 m_{22} + 2m_{11} m_{12} m_{21}\right) + 3E_2\left(m_{12} m_{21}^2 + 2m_{21} m_{22} m_{11}\right) \\ +3E_3 m_{21}^2 m_{22}\end{array}\right) \\ +\frac{1}{6}\left(W - B\right)\cos\theta 3 m_{41}^2 m_{42}\end{array}\right]$$

$$-\frac{a_{32}}{D}\left[\begin{array}{l}\frac{C_{D_y}}{6\gamma}\left(\begin{array}{l}3E_1 m_{11}^2 m_{12} + 3E_2\left(m_{11}^2 m_{22} + 2m_{11} m_{12} m_{21}\right) + 3E_3\left(m_{12} m_{21}^2 + 2m_{21} m_{22} m_{11}\right) \\ +3E_4 m_{21}^2 m_{22}\end{array}\right) \\ +\frac{1}{6}\left(x_G W - x_B B\right)\cos\theta 3 m_{41}^2 m_{42}\end{array}\right]$$

$$+\frac{a_{33}}{6D}\left(z_G W - z_B B\right)\cos\theta m_{41}^2 m_{42} \tag{40}$$

$$l_{33} = -\frac{a_{31}}{D}\left[\begin{array}{l} \frac{C_{D_Y}}{6\gamma}\left(\begin{array}{l} 3E_0 m_{11} m_{12}^3 + 3E_1\left(m_{12}^2 m_{21} + 2m_{11} m_{12} m_{22}\right) \\ + 3E_2\left(m_{22}^2 m_{11} + +2m_{21} m_{22} m_{12}\right) + 3E_3 m_{21} m_{22}^2 \end{array}\right) \\ + \frac{1}{6}(W - B)\cos\theta 3m_{41} m_{42}^2 \end{array}\right]$$

$$-\frac{a_{32}}{D}\left[\begin{array}{l} \frac{C_{D_Y}}{6\gamma}\left(\begin{array}{l} 3E_1 m_{11} m_{12}^2 + 3E_2\left(m_{12}^2 m_{21} + 2m_{11} m_{12} m_{22}\right) \\ + 3E_3\left(m_{22}^2 m_{11} + 2m_{21} m_{22} m_{12}\right) + 3E_4 m_{21} m_{22}^2 \end{array}\right) \\ + \frac{1}{6}(x_G W - x_B B)\cos\theta 3m_{41} m_{42}^2 \end{array}\right]$$

$$+\frac{a_{33}}{6D}(z_G W - z_B B)\cos\theta 3m_{41} m_{42}^2 \tag{41}$$

$$l_{34} = -\frac{a_{31}}{D}\left[\begin{array}{l} \frac{C_{D_Y}}{6\gamma}\left(E_0 m_{12}^3 + 3E_1 m_{11}^2 m_{21} + 3E_2 m_{12} m_{22}^2 + E_3 m_{22}^3\right) \\ + \frac{1}{6}(W - B)\cos\theta m_{42}^3 \end{array}\right]$$

$$+\frac{a_{32}}{D}\left[\begin{array}{l} \frac{C_{D_Y}}{6\gamma}\left(E_1 m_{12}^3 + 3E_2 m_{11}^2 m_{21} + 3E_3 m_{12} m_{22}^2 + E_4 m_{22}^3\right) \\ + \frac{1}{6}(x_G W - x_B B)\cos\theta m_{42}^3 \end{array}\right]$$

$$+\frac{a_{33}}{6D}(z_G W - z_B B)\cos\theta m_{42}^3 \tag{42}$$

The remaining procedure for determining the equation for the radius of the resulting limit cycles is identical to the one described in [Ref. 13, pg. 43, 44], which is:

$$\dot{R} = a'\varepsilon R + KR^3 \tag{43}$$

F. LIMIT CYCLE ANALYSIS

At steady state, $\dot{R} = 0$, equation (43) becomes:

$$0 = R\left(a'\varepsilon + KR^2\right) \tag{44}$$

Equation (66) has two solutions:

$$R = 0 \tag{45}$$

$$R = \sqrt{\frac{-a'\varepsilon}{K}} \tag{46}$$

Equation (45) is the trivial solution and gives no useful information. Equation (46) gives a constant amplitude limit cycle, R, in the cartesian coordinate system. This limit cycle will exist if the quotient inside the radical sign is positive:

$$\frac{-a'\varepsilon}{K} > 0 \tag{47}$$

This condition is necessary for R to be a real number. Since a' is always negative, the existence of limit cycles depends on the parameter K. The introduction of a nonzero pitch angle does not change the dependence the limit cycle has on the parameter K. Stated, this dependence is:

- If $K<0$, periodic solutions exist and they are stable.
- If $K>0$, periodic solutions exist and they are unstable.

G. RESULTS AND DISCUSSION

Results for the stability parameter, K, are presented in Figures 6 through 15 Figures 6 through 10 provide results for K at a constant forward speed, U, and varying pitch angles. The pitch angles were varied from positive 10 degrees to negative 10 degrees in 5 degree increments. In Figures 6 through 10, the bottom curve represents solutions for positive 10 degrees and the top curve represents negative 10 degrees. It is clear that all values of K are negative indicating they are stable solutions. Notice that decreasing pitch angles the solutions for K become less negative, indicating that while they are stable, these solutions are less stable than those at the higher pitch angles. In Figures 11 through 15, solutions for the stability parameter, K, are again represented, but with the pitch angle held constant and the forward speed, U, varied from 2 ft/sec to 8 ft/sec in 1.5 ft/sec increments. The bottom curve in Figures 11 through 15 represents $U =$ 2ft/sec. As forward speed increases, the curves representing the stability parameter K tend to become more negative, but in a more pronounced fashion than those where U was held constant.

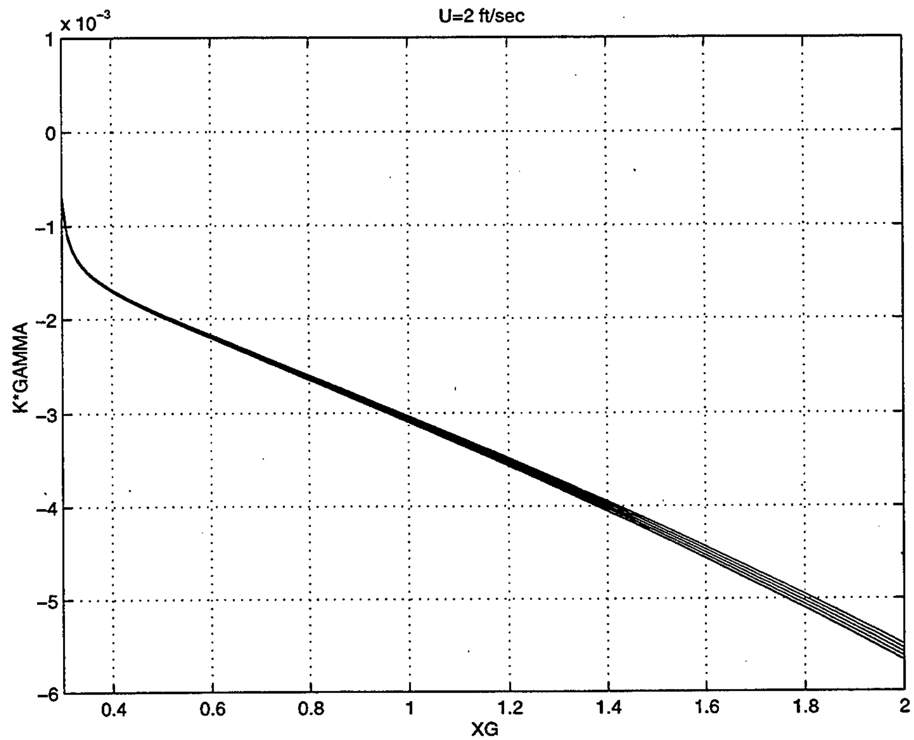

Figure 6: XG vs. K*GAMMA for U=2 ft/sec

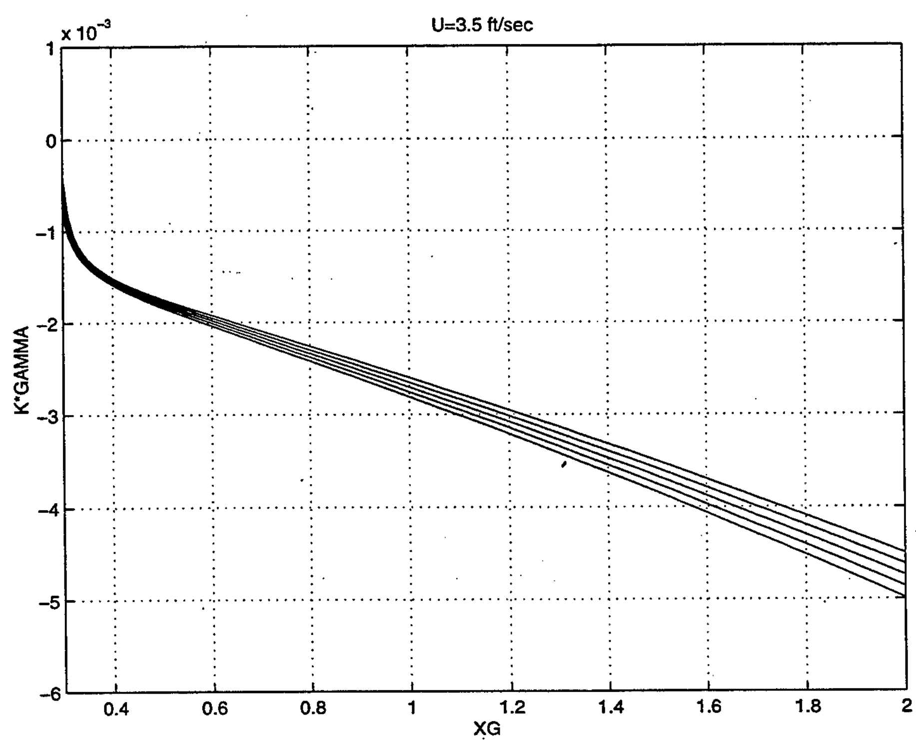

Figure 7: XG vs. K*GAMMA for U=3.5 ft/sec

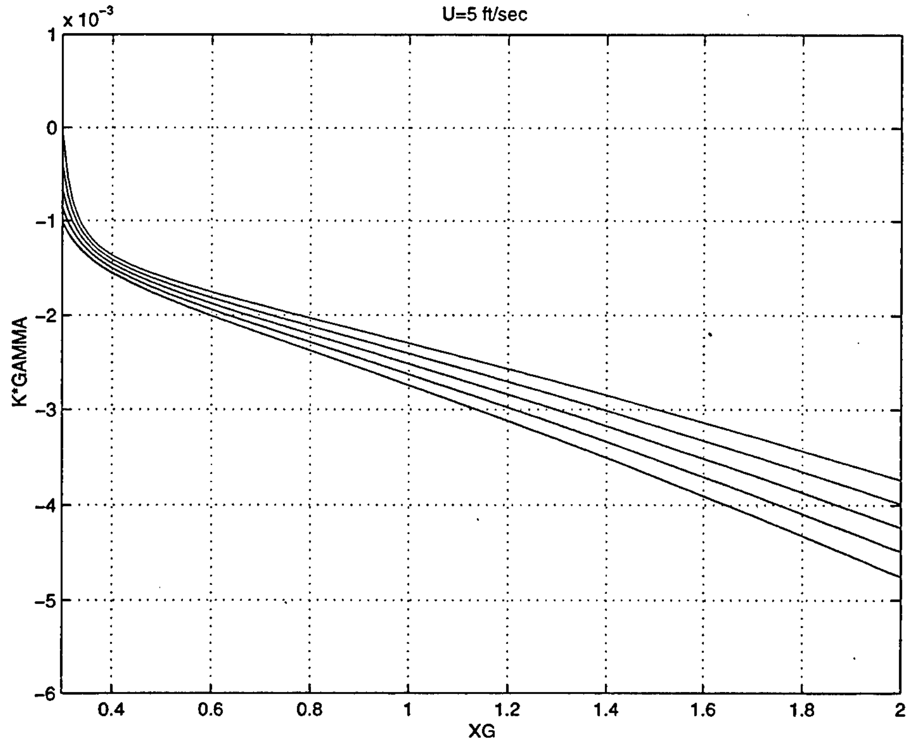

Figure 8: XG vs. K*GAMMA for U=5 ft/sec

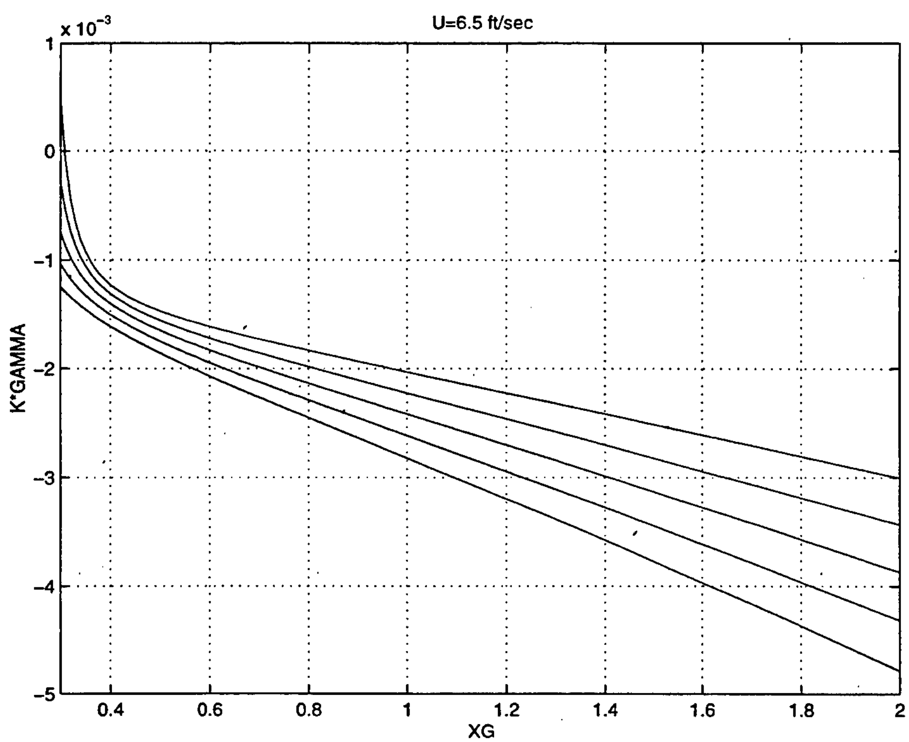

Figure 9: XG vs. K*GAMMA for U=6.5 ft/sec

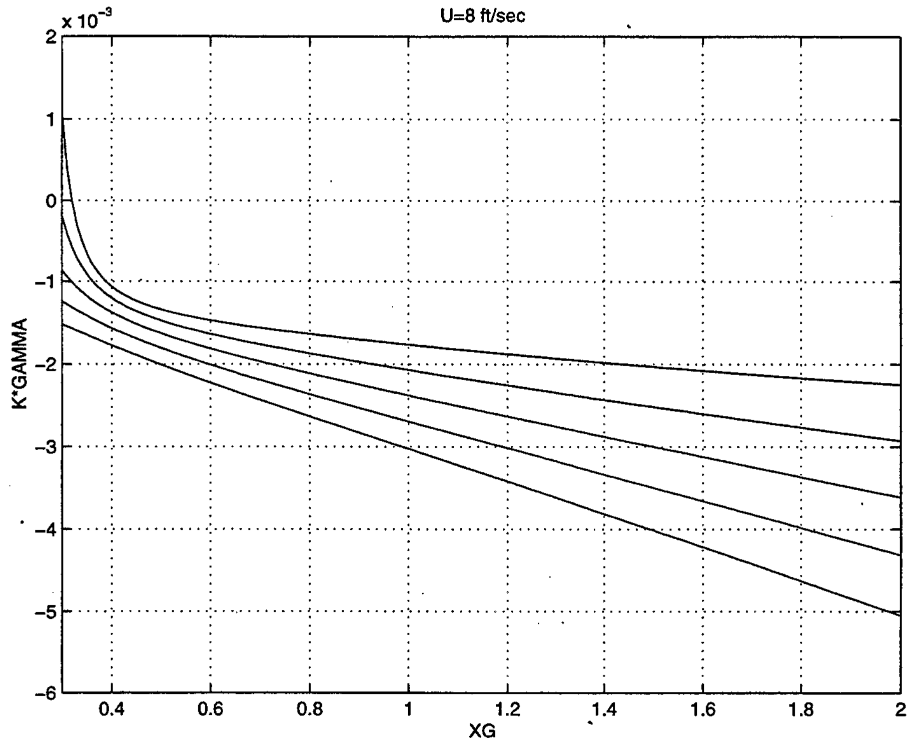

Figure 10: XG vs. K*GAMMA for U=8 ft/sec

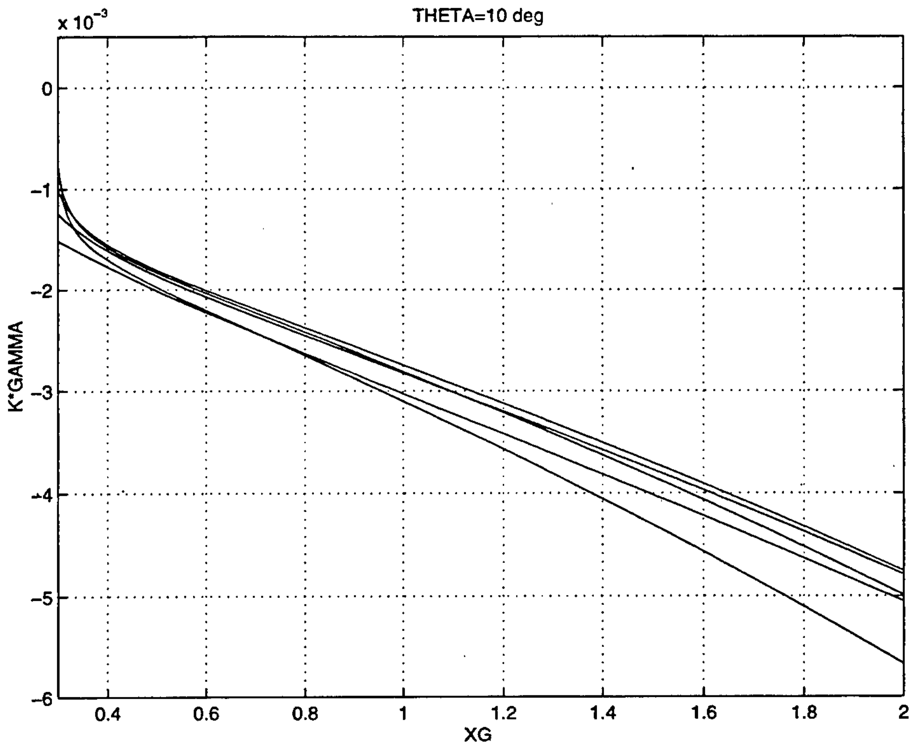

Figure 11: XG vs. K*GAMMA for THETA =10 deg

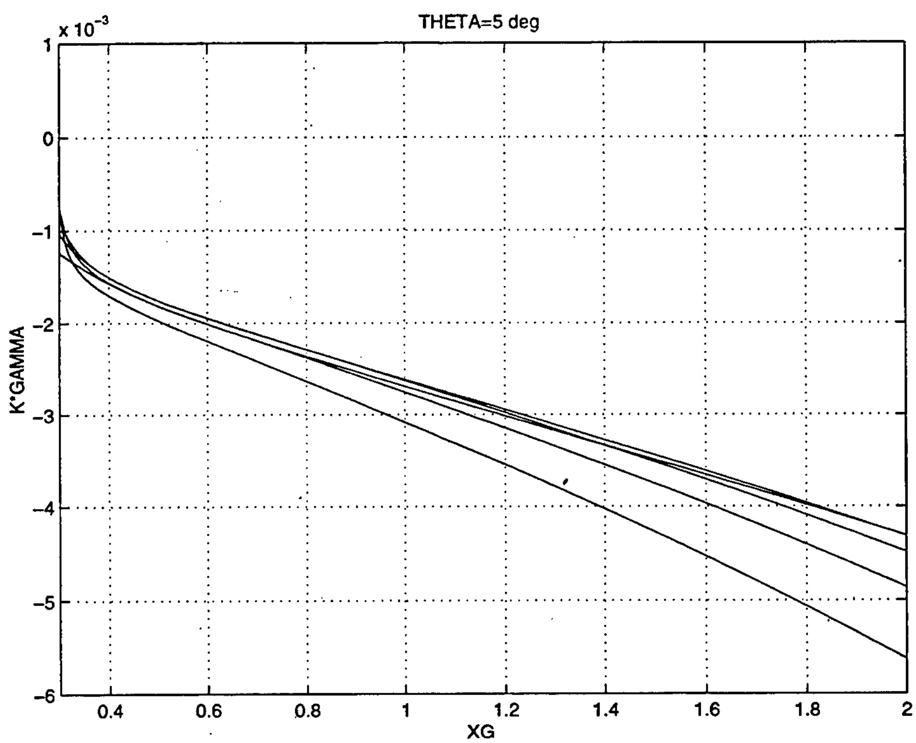

Figure 12: XG vs. K*GAMMA for THETA=5 deg

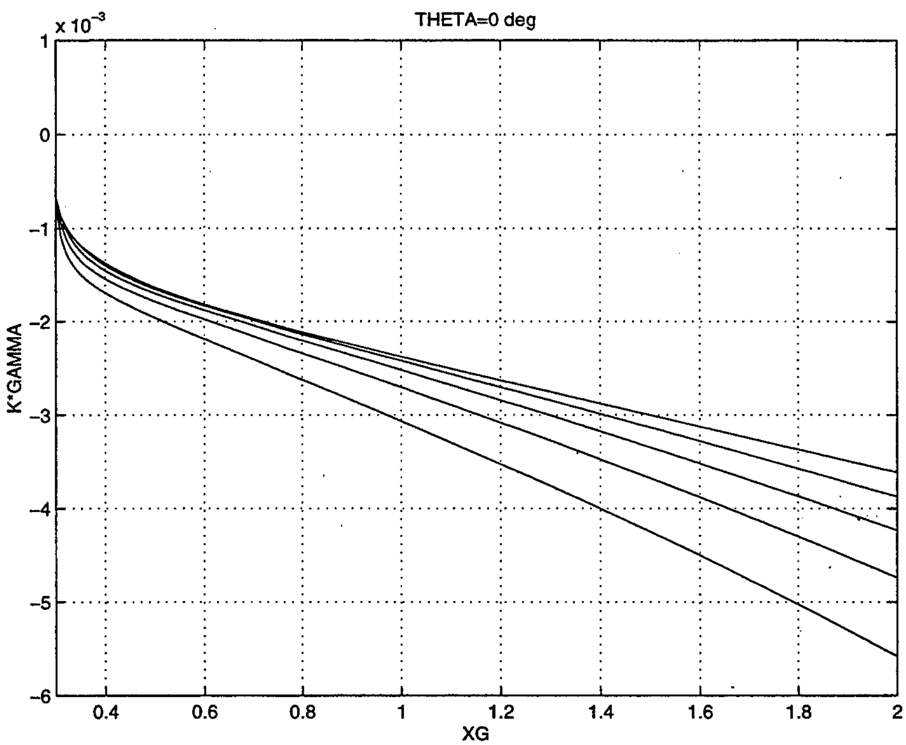

Figure 13: XG vs. K*GAMMA for THETA=0 deg

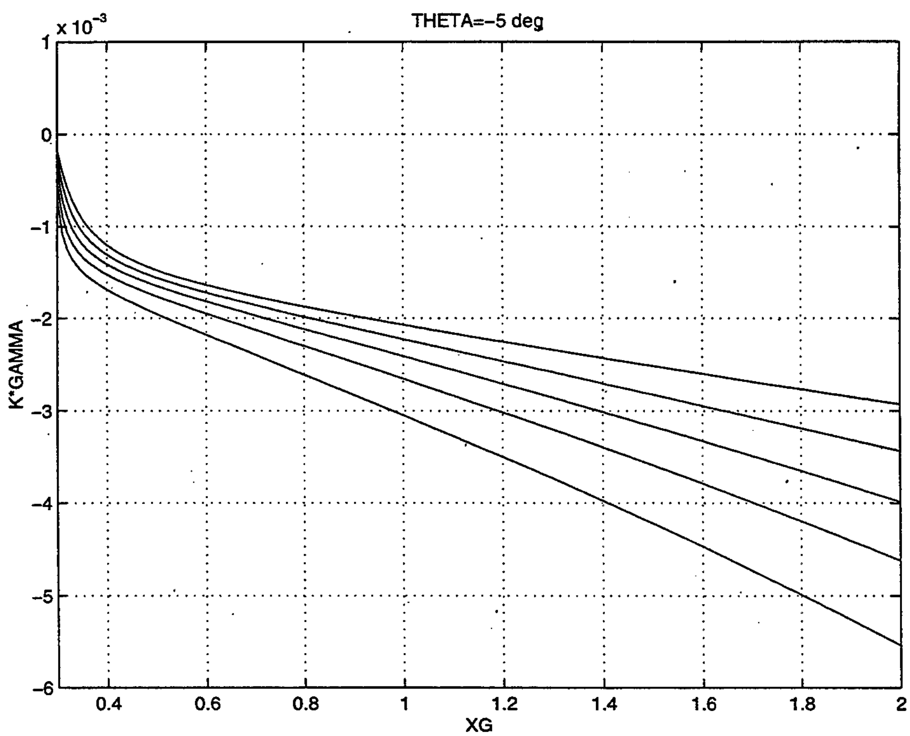

Figure 14: XG vs. K*GAMMA for THETA=-5 deg

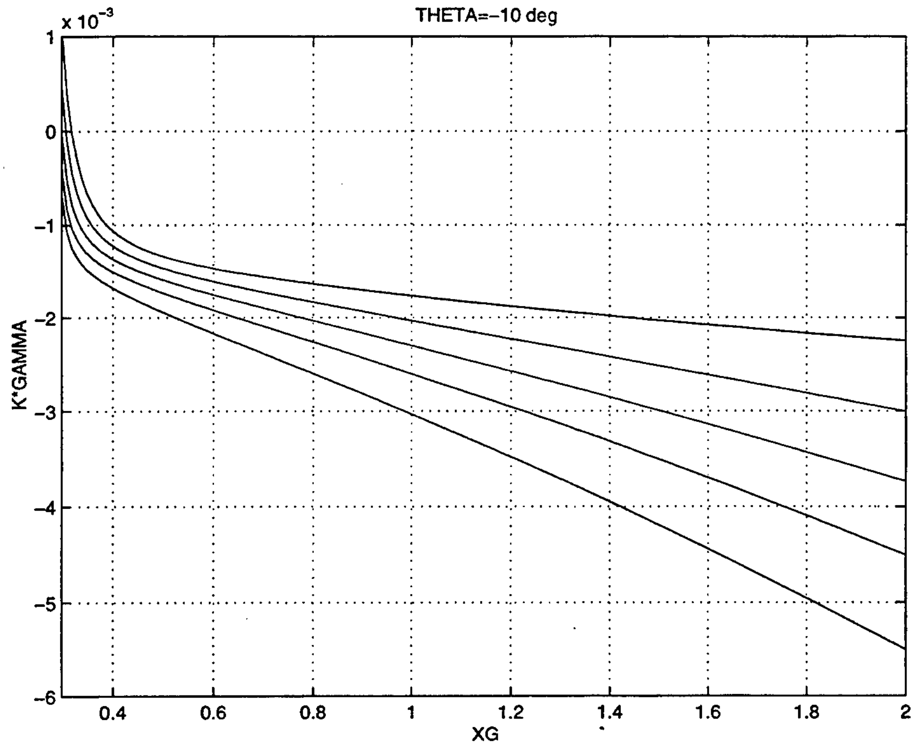

Figure 15: XG vs. K*GAMMA for THETA=-10 deg

V. CONCLUSIONS AND RECOMMENDATIONS

This thesis presented a continuing study of the formation of limit cycles due to the coupling of the sway/yaw/roll equations of motion. We have shown that loss of stability occurs in the form of stable limit cycles and that the addition of a nonzero pitch angle does not significantly affect the formation of these limit cycles. Through a linear analysis of the sway/yaw/roll equations of motion we demonstrated that the addition of a nonzero pitch angle affects the domain of stability of straight line motion, especially at higher speeds. This was validated by the nonlinear analysis as well. As a recommendation for further study in this area we suggest that the analysis be expanded to include coupling into the heave/pitch directions of motion as well as the effects automatic path control.

APPENDIX

The following is a list of the MATLAB and FORTRAN programs used in this thesis. Complete printouts of the programs accompany this list.

- **THESIS1.M**

  A MATLAB program for performing the linear analysis section of this thesis.

- **HOPF_1NEW.FOR**

  A FORTRAN program for performing the nonlinear analysis section of this thesis.

```
% THESIS1.M
%
% LOSS OF STABILITY
% ****************************************
clear

a=1;
W=12000;
IXX=1760; IYY=9450;
IZZ=10700; IXZ=0;  IXY=0;
IYZ=0; L=17.425; RHO=1.94;
G=32.2; U=8.0; M=W/G; B=W;
THETA=0;
THETA=THETA*pi/180;
OMEGA=U*tan(THETA);

ND1=0.5*RHO*L^2;

% DEFINE HYDRODYNAMIC COEFFICIENTS

YPDOT=1.270e-04*ND1*L^2;
YVDOT=-5.550e-02*ND1*L;
YRDOT=1.240e-03*ND1*L^2;
YP=3.055e-03*ND1*L;
YPOMEGA=0;
YP=YP*U+YPOMEGA*OMEGA+M*OMEGA;
YV=-9.310e-02*ND1;
YVOMEGA=0;
YV=YV+YVOMEGA*OMEGA;
YR=-5.940e-02*ND1*L;
YROMEGA=0;
YR=YR+YROMEGA*OMEGA;

NPDOT=-3.370e-05*ND1*L^3;
NVDOT=1.240e-03*ND1*L^2;
NRDOT=-3.400e-03*ND1*L^3;
NP=-8.405e-04*ND1*L^2;
NPOMEGA=0;
NP=NP+NPOMEGA*OMEGA;
NV=-1.484e-02*ND1*L;
NVOMEGA=0;
NV=NV+NVOMEGA*OMEGA;
NR=-1.640e-02*ND1*L^2;
NROMEGA=0;
NR=NR+NROMEGA*OMEGA;
```

```
KPDOT=-1.01e-03*ND1*L^3;
KVDOT=1.27e-04*ND1*L^2;
KRDOT=-3.37e-05*ND1*L^3;
KP=-1.10e-02*ND1*L^2;
KPOMEGA=0;
KP=KP+KPOMEGA*OMEGA;
KV=3.055e-03*ND1*L;
KVOMEGA=0;
KV=KV+KVOMEGA*OMEGA;
KR=-8.41e-04*ND1*L^2;
KROMEGA=0;
KR=KR+KROMEGA*OMEGA;

flag=0;
for XG=0:0.01:2,

flag=flag+1;

xg(flag)=XG;
a=IXX-KPDOT; b=KP*U; e=KV*U;
f=KRDOT; i=YP*U; j=M-YVDOT; k=YV*U;
l=XG*M-YRDOT; m=U*(YR-M); o=NPDOT;
p=NP*U; q=-XG*W; r=XG*M-NVDOT;
w=U*(NR-XG*M); x=NV*U; u=IZZ-NRDOT;

a1=-u*M^2;
a2=-u*M*YPDOT-u*M*KVDOT+M*o*l+f*r*M;
a3=a*j*u-a*l*r-u*KVDOT*YPDOT+KVDOT*o*l+f*r*YPDOT-f*o*j;

b1=(w*(M^2))+(r*U*(M^2));
b2=-M*e*u-M*u*i+w*M*YPDOT+w*KVDOT*M-o*m*M+p*l*M-
f*x*M+r*M*U*YPDOT-o*j*M*U+r*KR*U*M;
b3=-e*u*YPDOT+e*o*l-u*i*KVDOT+w*KVDOT*YPDOT-
KVDOT*o*m+KVDOT*p*l-a*j*w-a*k*u+a*l*x+a*r*m-b*j*u+b*l*r+f*r*i-
f*x*YPDOT+o*k*f-f*p*j+r*KR*U*YPDOT;

b2=b2+M*OMEGA*j*u-M*OMEGA*l*r;

c1=-x*(M^2)*U;
c2=r*i*M*U-x*M*U*YPDOT-x*KR*U*M+o*k*M*U-p*j*M*U+j*u*W-l*r*W-
q*l*M+w*i*M-m*p*M+e*w*M;
c3=a*k*w-a*m*x+b*j*w+b*k*u-b*l*x-b*r*m+r*i*KR*U-
x*KR*U*YPDOT+o*k*KR*U-p*j*KR*U-q*l*KVDOT+w*i*KVDOT-m*p*KVDOT-
e*u*i+e*w*YPDOT-e*o*m+e*p*l+f*q*j-f*x*i+f*p*k;
```

```
c2=c2-M*OMEGA*j*W-M*OMEGA*k*u-M*OMEGA*l*x+M*OMEGA*r*m;

d2=q*j*M*U-x*i*M*U+p*k*M*U+q*m*M-j*w*W-k*u*W+l*x*W+r*m*W;
d3=q*j*KR*U-x*i*KR*U+p*k*KR*U-f*q*k+q*m*KVDOT-e*q*l+e*w*i-e*m*p-
b*k*w+b*m*x;

d2=d2+M*OMEGA*k*w-M*OMEGA*U*(KR-M)*x;

e2=k*w*W-m*x*W-q*k*M*U;
e3=e*q*m-q*k*KR*U;

f5=(-e2*(b1^2))+b1*c1*d2;
f4=((b2*c1+b1*c2)*d2)+b1*c1*d3-a1*(d2^2)-e3*(b1^2)-2*e2*b1*b2;
f3=-a2*(d2^2)-2*d3*d2*a1-2*e3*b1*b2-
e2*((b2^2)+2*b1*b3)+d2*(b3*c1+b2*c2+b1*c3)+d3*(b2*c1+b1*c2);
f2=-e3*((b2^2)+2*b1*b3)-2*e2*b2*b3+d2*(b3*c2+b2*c3)+d3*(b3*c1+b2*c2+b1*c3)-
a3*(d2^2)-2*d3*d2*a2-a1*(d3^2);
f1=b3*c3*d2+d3*(b3*c2+b2*c3)-2*e3*b2*b3-e2*(b3^2)-2*d3*d2*a3-a2*(d3^2);
f0=b3*c3*d3-a3*(d3^2)-e3*(b3^2);

coef=[f5 f4 f3 f2 f1 f0];
ZG=roots(coef);

tot(flag)=ZG(5,1);
end
```

```
C   PROGRAM HOPF_1NEW.FOR
C   EVALUATION OF HOPF BIFURCATION FORMULAS USING THE SUBOFF
C   SUBMARINE MODEL
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      REAL*8 L,IYY,M,YPDOT,YVDOT,ND1,YRDOT
      REAL*8 YP,YV,YR,NPDOT,NVDOT,NRDOT,NP,NV,NR
      REAL*8 GAMA,U,KVDOT,KRDOT,KPDOT,D
      REAL*8 E0,E1,E2,E3,E4,XG,ZG,KR,KP,KV,XG1,ZG1
C
      REAL*8 M11,M12,M13,M14,M21,M22,M23
      REAL*8 M24,M31,M32,M33,M34,M41,M42,M43,M44
      REAL*8 N11,N12,N13,N14,N21,N22,N23,N24
      REAL*8 N31,N32,N33,N34,N41,N42,N43,N44
      REAL*8 L11,L12,L13,L14,L21,L22,L23,L24,L31
      REAL*8 L32,L33,L34,L15,L16,L17,L25,L26,L27,L35
      REAL*8 L36,L37,L1A,L2A,L3A,L4A,L5A,L6A,L7A,L8A,L9A
      REAL*8 L10A,L11A,L12A,L1,L2,L3,L4,L5,L6,L7
      REAL*8 L8,L9,R11,R12,R13,R14,R21,R22,R23,R24
      REAL*8 P11,P12,P13,P21,P22,P23
      DOUBLE PRECISION THETA
C
      DIMENSION F(4,4),T(4,4),TINV(4,4),FV1(4),IV1(4),YYY(4,4)
      DIMENSION WR(4),WI(4),TSAVE(4,4),TLUD(4,4),IVLUD(4),SVLUD(4)
      DIMENSION ASAVE(4,4),A2(4,4),XL(18),HT(18),ZGG(202),FF(4,4)
      DIMENSION VEC0(18),VEC1(18),VEC2(18),VEC3(18),VEC4(18),XGG(202)
C
      INTEGER I,J,K
C
      OPEN (20,FILE='HOPF25.RES')
      OPEN (21,FILE='DAT25.DAT',STATUS='OLD')
C      OPEN (23,FILE='HOPF1.RES',STATUS='OLD')
C
      WEIGHT=12000.0
      IXX  =1760.0
      IYY  =9450.0
      IZZ  =10700.0
      IXZ  =0.0
      IXY  =0.0
      IYZ  =0.0
      L    =17.425
      RHO  =1.94
      CD   =0.5
C     CD   =0.5*CD*RHO  ???
      G    =32.2
```

```
      XB   =0.0
      ZB   =0.0
      YG   =0.0
      YB   =0.0
      YDELTAR=0.0
      DELTAR=0.0
      NDELTAR=0.0
      NPROP=0.0
      M  = WEIGHT/G
      B  = WEIGHT
      W=WEIGHT
         WRITE (*,*) ' ENTER U'
         READ (*,*) U
         WRITE (*,*) ' ENTER THETA (DEGREES)'
         READ (*,*) THETA
C      U  =5.0
      ND1  = 0.5*RHO*L**2
C      THETA=5
      THETA=THETA*PI/180
      OMEGA=U*DTAN(THETA)
C
C   DEFINE HYDRODYNAMIC COEFFICIENTS
      YPDOT=1.270E-04*ND1*L**2
      YVDOT=-5.550E-02*ND1*L
      YRDOT=1.240E-03*ND1*L**2
      YP=3.055E-03*ND1*L
      YPOMEGA=0.D0
      YP=YP+YPOMEGA*OMEGA+M*OMEGA
      YV=-9.310E-02*ND1
      YVOMEGA=0.D0
      YV=YV+YVOMEGA*OMEGA
      YR=-5.940E-02*ND1*L
      YROMEGA=0.D0
      YR=YR+YROMEGA*OMEGA
C
      NPDOT=-3.370E-05*ND1*L**3
      NVDOT=1.240E-03*ND1*L**2
      NRDOT=-3.400E-03*ND1*L**3
      NP=-8.405E-04*ND1*L**2
      NPOMEGA=0.D0
      NP=NP+NPOMEGA*OMEGA
      NV=-1.484E-02*ND1*L
      NVOMEGA=0.D0
      NV=NV+NVOMEGA*OMEGA
      NR=-1.640E-02*ND1*L**2
```

```
      NROMEGA=0.D0
      NR=NR+NROMEGA*OMEGA
C
      KPDOT=-1.01E-03*ND1*L**3
      KVDOT=1.27E-04*ND1*L**2
      KRDOT=-3.37E-05*ND1*L**3
      KP=-1.10E-02*ND1*L**2
      KPOMEGA=0.D0
      KP=KP+KPOMEGA*OMEGA
      KV=3.055E-03*ND1*L
      KVOMEGA=0.D0
      KV=KV+KVOMEGA*OMEGA
      KR=-8.41E-04*ND1*L**2
      KROMEGA=0.D0
      KR=KR+KROMEGA*OMEGA
C
C     DEFINE THE LENGTH X AND HEIGHT H TERMS FOR THE INTEGRATION
C     ALL IN FEET.
      XL( 1)=-105.9/12.0
      XL( 2)=-104.3/12.0
      XL( 3)=-99.3/12.0
      XL( 4)=-94.3/12.0
      XL( 5)=-87.3/12.0
      XL( 6)=-76.8/12.0
      XL( 7)=-66.3/12.0
      XL( 8)=-55.8/12.0
      XL( 9)=72.7/12.0
      XL(10)=79.2/12.0
      XL(11)=83.2/12.0
      XL(12)=87.2/12.0
      XL(13)=91.2/12.0
      XL(14)=95.2/12.0
      XL(15)=99.2/12.0
      XL(16)=101.2/12.0
      XL(17)=102.1/12.0
      XL(18)=103.2/12.0
C
      HT( 1)= 0.000
      HT( 2)= 2.28/12.0
      HT( 3)= 8.24/12.0
      HT( 4)= 13.96/12.0
      HT( 5)= 19.76/12.0
      HT( 6)= 25.1/12.0
      HT( 7)= 29.36/12.0
      HT( 8)= 31.85/12.0
```

```
      HT( 9)= 31.85/12.0
      HT(10)= 30.00/12.0
      HT(11)= 27.84/12.0
      HT(12)= 25.12/12.0
      HT(13)= 21.44/12.0
      HT(14)= 17.12/12.0
      HT(15)= 12.0/12.0
      HT(16)= 9.12/12
      HT(17)= 6.72/12
      HT(18)= 0.00
C
      DO 104 K = 1,18
        VEC0(K)=HT(K)
        VEC1(K)=XL(K)*HT(K)
        VEC2(K)=XL(K)*XL(K)*HT(K)
        VEC3(K)=XL(K)*XL(K)*XL(K)*HT(K)
        VEC4(K)=XL(K)*XL(K)*XL(K)*XL(K)*HT(K)
  104 CONTINUE
      CALL TRAP(18,VEC0,XL,E0)
      CALL TRAP(18,VEC1,XL,E1)
      CALL TRAP(18,VEC2,XL,E2)
      CALL TRAP(18,VEC3,XL,E3)
      CALL TRAP(18,VEC4,XL,E4)
C
      GAMA=0.001
C=================================================================
============
C     READ THE CRITICAL VALUES FOR XG AND ZG FROM FILE DATA1.DAT
C
        XGG(1)=0.0
        ZGG(1)=0.016358083
      DO 1 I=2,202
        READ (21,*)XG,ZG
C         WRITE(*,*)XG,ZG
        XGG(I)=XG
        ZGG(I)=ZG
C=================================================================
============
C     DETERMINE [F] COEFFICIENTS
C
      D=((M-YVDOT)*(IZZ-NRDOT)*(IXX-KPDOT))
     & -((M-YVDOT)*(IXZ-KRDOT)*(-IXZ-NPDOT))
     & -((M*XG-NVDOT)*(M*XG-YRDOT)*(IXX-KPDOT))
     & +((M*XG-NVDOT)*(IXZ-KRDOT)*(-M*ZG-YPDOT))
     & +((-M*ZG-KVDOT)*(M*XG-YRDOT)*(-IXZ-NPDOT))
```

```
     & -((-M*ZG-KVDOT)*(IZZ-NRDOT)*(-M*ZG-YPDOT))
C
      A11=((IZZ-NRDOT)*(IXX-KPDOT))-((IXZ-KRDOT)*(-IXZ-NPDOT))
      A12=((-M*XG+YRDOT)*(IXX-KPDOT))+((IXZ-KRDOT)*(-M*ZG-YPDOT))
      A13=((M*XG-YRDOT)*(-IXZ-NPDOT))-((IZZ-NRDOT)*(-M*ZG-YPDOT))
      A21=((-M*XG+NVDOT)*(IXX-KPDOT))+((-M*ZG-KVDOT)*(-IXZ-NPDOT))
      A22=((M-YVDOT)*(IXX-KPDOT))-((-M*ZG-KVDOT)*(-M*ZG-YPDOT))
      A23=((-M+YVDOT)*(-IXZ-NPDOT))+((M*XG-NVDOT)*(-M*ZG-YPDOT))
      A31=((M*XG-NVDOT)*(IXZ-KRDOT))-((-M*ZG-KVDOT)*(IZZ-NRDOT))
      A32=((-M+YVDOT)*(IXZ-KRDOT))+((-M*ZG-KVDOT)*(M*XG-YRDOT))
      A33=((M-YVDOT)*(IZZ-NRDOT))-((M*XG-NVDOT)*(M*XG-YRDOT))
C=================================================================
===
C     EVALUATE TRANSFORMATION MATRIX OF EIGENVECTORS
C
      F(1,1)=(A11*YV*U+A12*NV*U+A13*KV*U)/D
      F(1,2)=(A11*(YR*U-M*U)+A12*(-M*XG*U+NR*U)+A13*(M*ZG*U+KR*U))/D
      F(1,3)=(A11*YP*U+A12*NP*U+A13*(KP*U-M*ZG*OMEGA))/D
      F(1,4)=(A11*(W-B)*DCOS(THETA)+A12*XG*(W-XB)*B*DCOS(THETA)+
     &A13*(-ZG*W+ZB)*B*DCOS(THETA))/D
      F(2,1)=(A21*YV*U+A22*NV*U+A23*KV*U)/D
      F(2,2)=(A21*(YR*U-M*U)+A22*(-M*XG*U+NR*U)+A23*(M*ZG*U+KR*U))/D
      F(2,3)=(A21*YP*U+A22*NP*U+A23*(KP*U-M*ZG*OMEGA))/D
      F(2,4)=(A21*(W-B)*DCOS(THETA)+A22*(XG*W-XB*B)*DCOS(THETA)+
     &A23*(-ZG*W+ZB*B)*DCOS(THETA))/D
      F(3,1)=(A31*YV*U+A32*NV*U+A33*KV*U)/D
      F(3,2)=(A31*(YR*U-M*U)+A32*(-M*XG*U+NR*U)+A33*(M*ZG*U+KR*U))/D
      F(3,3)=(A31*YP*U+A32*NP*U+A33*(KP*U-M*ZG*OMEGA))/D
      F(3,4)=(A31*(W-B)*DCOS(THETA)+A32*(XG*W-XB*B)*DCOS(THETA)+
     &A33*(-ZG*W+ZB*B)*DCOS(THETA))/D
      F(4,1)=0.0
      F(4,2)=0.0
      F(4,3)=1.0
      F(4,4)=0.0
C
      DO 11 K=1,4
       DO 12 J=1,4
        ASAVE(K,J)=F(K,J)
 12     CONTINUE
 11   CONTINUE
      CALL RG(4,4,F,WR,WI,1,YYY,IV1,FV1,IERR)
      WRITE(23,1007)WR(1),WR(2),WR(3),WR(4)
      CALL DSOMEG(IEV,WR,WI,OMEGA,CHECK)
C     WRITE (*,*) CHECK
C     WRITE (*,*) WR(2)
```

```
C      WRITE (*,*) WI(4)
      OMEGA0=OMEGA
      DO 5 J=1,4
       T(J,1)= YYY(J,IEV)
       T(J,2)=-YYY(J,IEV+1)
    5 CONTINUE
      IF (IEV.EQ.1.0) GO TO 13
      IF (IEV.EQ.2.0) GO TO 18
      IF (IEV.EQ.3.0) GO TO 14
C      STOP 3004
   14  DO 6 J=1,4
       T(J,3)=YYY(J,1)
       T(J,4)=YYY(J,2)
    6 CONTINUE
      GO TO 17
   18  DO 19 J=1,4
       T(J,3)=YYY(J,1)
       T(J,4)=YYY(J,4)
   19  CONTINUE
      GO TO 17
   13  DO 16 J=1,4
       T(J,3)=YYY(J,3)
       T(J,4)=YYY(J,4)
   16  CONTINUE
   17  CONTINUE
C
C     NORMALIZATION OF THE CRITICAL EIGENVECTOR
C
      CALL NORMAL(T)
C
C     INVERT TRANSFORMATION MATRIX
C
      DO 2 K=1,4
       DO 3 J=1,4
        TINV(K,J)=0.0
        TSAVE(K,J)=T(K,J)
    3   CONTINUE
    2 CONTINUE
      CALL DLUD(4,4,TSAVE,4,TLUD,IVLUD)
C     DO 4 J=1,4
C        IF (IVLUD(J).EQ.0) STOP 3003
C   4  CONTINUE
      CALL DILU(4,4,TLUD,IVLUD,SVLUD)
      DO 8 K=1,4
       DO 9 J=1,4
```

```
      TINV(K,J)=TLUD(K,J)
 9    CONTINUE
 8   CONTINUE
C
C     CHECK Inv(T)*A*T
C
     CALL MULT(TINV,ASAVE,T,A2)
C
     P1=A2(3,3)
     P2=A2(4,4)
     PEIG1=P1
     PEIG2=P2
C
C     DEFINITION OF Nij
C
    N11=TINV(1,1)
    N12=TINV(1,2)
    N13=TINV(1,3)
    N14=TINV(1,4)
    N21=TINV(2,1)
    N22=TINV(2,2)
    N23=TINV(2,3)
    N24=TINV(2,4)
    N31=TINV(3,1)
    N32=TINV(3,2)
    N33=TINV(3,3)
    N34=TINV(3,4)
    N41=TINV(4,1)
    N42=TINV(4,2)
    N43=TINV(4,3)
    N44=TINV(4,4)
C
C     DEFINITION OF Mij
C
    M11=T(1,1)
    M12=T(1,2)
    M13=T(1,3)
    M14=T(1,4)
    M21=T(2,1)
    M22=T(2,2)
    M23=T(2,3)
    M24=T(2,4)
    M31=T(3,1)
    M32=T(3,2)
    M33=T(3,3)
```

```
      M34=T(3,4)
      M41=T(4,1)
      M42=T(4,2)
      M43=T(4,3)
      M44=T(4,4)
C
C     DEFINITION OF Lij
C
      L1=YG*((M31**2)+(M21**2))
      L2=IXY*(M31**2)+IYZ*M31*M21+YG*M21*M11
      L3=IXY*M31*M21+IYZ*(M21**2)+M*YG*M11*M31+YG*W*(M41**2)-
YB*B*(M41**
     &2)
      L4=YG*(2.0*M31*M32+2.0*M21*M22)

L5=2.0*IXY*M31*M32+IYZ*(M31*M22+M32*M21)+YG*(M21*M12+M22*M11)

L6=IXY*(M31*M22+M32*M21)+2.0*IYZ*M21*M22+M*YG*(M11*M32+M12*M31
)+2.
     &0*(YG*W-YB*B)*M41*M42
      L7=YG*((M32**2)+(M22**2))
      L8=IXY*(M32**2)+IYZ*M32*M22+YG*M22*M12
      L9=IXY*M32*M22+IYZ*(M22**2)+M*YG*M12*M32+(YG*W-
YB*B)*(M42**2)
C
C
      L15=(A11/D)*L1+(A12/D)*L2-(A13/D)*L3
      L16=(A11/D)*L4+(A12/D)*L5-(A13/D)*L6
      L17=(A11/D)*L7+(A12/D)*L8-(A13/D)*L9
      L25=(A21/D)*L1+(A22/D)*L2-(A23/D)*L3
      L26=(A21/D)*L4+(A22/D)*L5-(A23/D)*L6
      L27=(A21/D)*L7+(A22/D)*L8-(A23/D)*L9
      L35=(A31/D)*L1+(A32/D)*L2-(A33/D)*L3
      L36=(A31/D)*L4+(A32/D)*L5-(A33/D)*L6
      L37=(A31/D)*L7+(A32/D)*L8-(A33/D)*L9
C
      C=CD/(6.0*GAMA)

L1A=C*(E0*(M11**3)+3.0*E1*(M11**2)*M21+3.0*E2*M11*(M21**2)+E3*(M21
     &**3))-((1.0/6.0)*(W-B)*DCOS(THETA))*(M41**3)

L2A=C*(E1*(M11**3)+3.0*E2*(M11**2)*M21+3.0*E3*M11*(M21**2)+E4*(M21
     &**3))-((1.0/6.0)*(XG*W-XB*B)*DCOS(THETA))*(M41**3)
      L3A=((1.0/6.0)*(ZG*W-ZB*B)*DCOS(THETA))*(M41**3)
```

```
L4A=C*(3.0*E0*(M11**2)*M12+3.0*E1*((M11**2)*M22+2*M11*M12*M21)+3.0
   &*E2*(M12*(M21**2)+2.0*M21*M22*M11)+3.0*E3*(M21**2)*M22)-((1.0/6.0)
   &*(W-B)*DCOS(THETA))*3.0*(M41**2)*M42

L5A=C*(3.0*E1*(M11**2)*M12+3.0*E2*((M11**2)*M22+2*M11*M12*M21)+3.0
   &*E3*(M12*(M21**2)+2*M21*M22*M11)+3.0*E4*(M21**2)*M22)-((1.0/6.0)*(X
   &G*W-XB*B)*DCOS(THETA))*3.0*(M41**2)*M42
    L6A=((1.0/6.0)*(ZG*W-ZB*B)*DCOS(THETA))*3.0*(M41**2)*M42

L7A=C*(3*E0*M11*(M12**2)+3.0*E1*((M12**2)*M21+2*M11*M12*M22)+3.0*E
   &2*((M22**2)*M11+2.0*M21*M22*M12)+3.0*E3*M21*(M22**2))-((1.0/6.0)*(W
   &-B)*DCOS(THETA))*3.0*M41*(M42**2)

L8A=C*(3.0*E1*M11*(M12**2)+3.0*E2*((M12**2)*M21+2.0*M11*M12*M22)+3
   &.0*E3*((M22**2)*M11+2*M21*M22*M12)+3.0*E4*M21*(M22**2))
    L9A=((1.0/6.0)*(ZG*W-ZB*B)*DCOS(THETA))*3.0*M41*(M42**2)
    L10A=-C*(E0*(M12**3)+3.0*E1*(M11**2)*M21+3.0*E2*M12*(M22**2)+E3*(M
   &22**3))-((1.0/6.0)*(W-B)*DCOS(THETA))*(M42**3)
    L11A=-C*(E1*(M12**3)+3.0*E2*(M11**2)*M21+3.0*E3*M12*(M22**2)+E4*(M
   &22**3))-((1.0/6.0)*(XG*W-XB*B)*DCOS(THETA))*(M42**3)
    L12A=((1.0/6.0)*(ZG*W-ZB*B)*DCOS(THETA))*(M42**3)
C
    L11=(-A11/D)*L1A+(A12/D)*L2A+(A13/D)*L3A
    L12=(-A11/D)*L4A+(A12/D)*L5A+(A13/D)*L6A
    L13=(-A11/D)*L7A+(A12/D)*L8A+(A13/D)*L9A
    L14=(-A11/D)*L10A+(A12/D)*L11A+(A13/D)*L12A
C
    L21=(-A21/D)*L1A+(A22/D)*L2A+(A23/D)*L3A
    L22=(-A21/D)*L4A+(A22/D)*L5A+(A23/D)*L6A
    L23=(-A21/D)*L7A+(A22/D)*L8A+(A23/D)*L9A
    L24=(-A21/D)*L10A+(A22/D)*L11A+(A23/D)*L12A
C
    L31=(-A31/D)*L1A+(A32/D)*L2A+(A33/D)*L3A
    L32=(-A31/D)*L4A+(A32/D)*L5A+(A33/D)*L6A
    L33=(-A31/D)*L7A+(A32/D)*L8A+(A33/D)*L9A
    L34=(-A31/D)*L10A+(A32/D)*L11A+(A33/D)*L12A
C
    R11=(N11*L11)+(N12*L21)+(N13*L31)
    R12=(N11*L12)+(N12*L22)+(N13*L32)
    R13=(N11*L13)+(N12*L23)+(N13*L33)
    R14=(N11*L14)+(N12*L24)+(N13*L34)
    R21=(N21*L11)+(N22*L21)+(N23*L31)
    R22=(N21*L12)+(N22*L22)+(N23*L32)
    R23=(N21*L13)+(N22*L23)+(N23*L33)
```

```
      R24=(N21*L14)+(N22*L24)+(N23*L34)
C
      P11=(N11*L15)+(N12*L25)+(N13*L35)
      P12=(N11*L16)+(N12*L26)+(N13*L36)
      P13=(N11*L17)+(N12*L27)+(N13*L37)
      P21=(N21*L15)+(N22*L25)+(N23*L35)
      P22=(N21*L16)+(N22*L26)+(N23*L36)
      P23=(N21*L17)+(N22*L27)+(N23*L37)
C
C     EVALUATE DALPHA AND DOMEGA
C
       ZGR =ZGG(I)
       ZGL =ZGG(I-1)
       ZG1 =ZGR
       XG1 =XGG(I)
C
       CALL FMATRIX(ZG1,XG1,FF,U,THETA)
C
       CALL RG(4,4,FF,WR,WI,0,YYY,IV1,FV1,IERR)
       CALL DSTABL(DEOS,WR,WI,FREQ)
       ALPHR=DEOS
       OMEGR=FREQ
C
       ZG1  =ZGL
       XG1  =XGG(I-1)
C
       CALL FMATRIX(ZG1,XG1,FF,U,THETA)
C
       CALL RG(4,4,FF,WR,WI,0,YYY,IV1,FV1,IERR)
       CALL DSTABL(DEOS,WR,WI,FREQ)
       ALPHL=DEOS
       OMEGL=FREQ
C
       DALPHA=(ALPHR-ALPHL)/(ZGR-ZGL)
       DOMEGA=(OMEGR-OMEGL)/(ZGR-ZGL)
C
C      EVALUATION OF HOPF BIFURCATION COEFFICIENTS
C
      COEF1=(1.0/8.0)*(3.0*R11+R13+R22+3.0*R24)
      COEF2=(1.0/8.0)*(3.0*R11+R23-R12-3.0*R14)

C      AMU2 =-COEF1/(8.0*DALPHA)   ????????
C      BETA2=0.25*COEF1            ???????
C      TAU2 =-(COEF2-DOMEGA*COEF1/DALPHA)/(8.0*OMEGA0)
C      PER  =2.0*3.1415927/OMEGA0
```

```
C      PER =PER*U/L
      WRITE (20,2001)XG,ZG,COEF1,DALPHA,OMEGA0,PEIG1,PEIG2
C      WRITE (20,2001)COEF1
  1 CONTINUE
C
      STOP

 2001 FORMAT (7E14.5)
 4001 FORMAT (3F15.5)
 1007 FORMAT (4E14.5)
        END
```

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